Chapter 2
Functions and Their Graphs

Section 2.1

1. \((-1,3)\)

3. We must not allow the denominator to be 0.
   \(x + 4 \neq 0 \Rightarrow x \neq -4\); Domain: \(\{x \mid x \neq -4\}\).

5. independent; dependent

7. \([0,5]\)
   We need the intersection of the intervals \([0,7]\) and \([-2,5]\).

9. \(g(x) - f(x)\), or \((g - f)(x)\)

11. True

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of \(f\) is a real number.

15. Function
   Domain: \{Elvis, Colleen, Kaleigh, Marissa\}
   Range: \{Jan. 8, Mar. 15, Sept 17\}

17. Not a function

19. Not a function

21. Function
   Domain: \{1, 2, 3, 4\}
   Range: \{3\}

23. Not a function

25. Function
   Domain: \{-2, -1, 0, 1\}
   Range: \{0, 1, 4\}

27. Graph \(y = x^2\). The graph passes the vertical line test. Thus, the equation represents a function.

29. Graph \(y = \frac{1}{x}\). The graph passes the vertical line test. Thus, the equation represents a function.

31. \(y^2 = 4 - x^2\)
   Solve for \(y\): \(y = \pm\sqrt{4 - x^2}\)
   For \(x = 0\), \(y = \pm 2\). Thus, \((0, 2)\) and \((0, -2)\) are on the graph. This is not a function, since a distinct \(x\) corresponds to two different \(y\)'s.

33. \(x = y^2\)
   Solve for \(y\): \(y = \pm\sqrt{x}\)
   For \(x = 1\), \(y = \pm 1\). Thus, \((1, 1)\) and \((1, -1)\) are on the graph. This is not a function, since a distinct \(x\) corresponds to two different \(y\)'s.

35. Graph \(y = 2x^2 - 3x + 4\). The graph passes the vertical line test. Thus, the equation represents a function.
37. \(2x^2 + 3y^2 = 1\)
Solve for \(y\): \(2x^2 + 3y^2 = 1\)
\[3y^2 = 1 - 2x^2\]
\[y^2 = \frac{1 - 2x^2}{3}\]
\[y = \pm \sqrt{\frac{1 - 2x^2}{3}}\]
For \(x = 0\), \(y = \pm \frac{1}{\sqrt{3}}\). Thus, \(0, \frac{1}{\sqrt{3}}\) and \(0, -\frac{1}{\sqrt{3}}\) are on the graph. This is not a function, since a distinct \(x\) corresponds to two different \(y\)'s.

39. \(f(x) = 3x^2 + 2x - 4\)
   a. \(f(0) = 3(0)^2 + 2(0) - 4 = -4\)
   b. \(f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1\)
   c. \(f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3\)
   d. \(f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4\)
   e. \(-f(x) = -(3x^2 + 2x - 4) = -3x^2 - 2x + 4\)
   f. \(f(x + 1) = 3(x + 1)^2 + 2(x + 1) - 4\)
      \[= 3(x^2 + 2x + 1) + 2x + 2 - 4\]
      \[= 3x^2 + 6x + 3 + 2x + 2 - 4\]
      \[= 3x^2 + 8x + 1\]
   g. \(f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4\)
   h. \(f(x + h) = 3(x + h)^2 + 2(x + h) - 4\)
      \[= 3(x^2 + 2xh + h^2) + 2x + 2h - 4\]
      \[= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4\]
41. \(f(x) = \frac{x}{x^2 + 1}\)
   a. \(f(0) = \frac{0}{0^2 + 1} = 0\)
   b. \(f(1) = \frac{1}{1^2 + 1} = \frac{1}{2}\)

43. \(f(x) = |x| + 4\)
   a. \(f(0) = |0| + 4 = 0 + 4 = 4\)
   b. \(f(1) = |1| + 4 = 1 + 4 = 5\)
   c. \(f(-1) = |-1| + 4 = 1 + 4 = 5\)
   d. \(f(-x) = |-x| + 4 = |x| + 4\)
   e. \(-f(x) = -(|x| + 4) = -|x| - 4\)
   f. \(f(x + 1) = |x + 1| + 4\)
   g. \(f(2x) = 2|x| + 4 = 2|x| + 4\)
   h. \(f(x + h) = |x + h| + 4\)

45. \(f(x) = \frac{2x + 1}{3x - 5}\)
   a. \(f(0) = \frac{2(0) + 1}{3(0) - 5} = \frac{0 + 1}{0 - 5} = \frac{-1}{5}\)
   b. \(f(1) = \frac{2(1) + 1}{3(1) - 5} = \frac{2 + 1}{3 - 5} = \frac{3}{-2} = -\frac{3}{2}\)
   c. \(f(-1) = \frac{2(-1) + 1}{3(-1) - 5} = \frac{-2 + 1}{-3 - 5} = \frac{-1}{8}\)
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Chapter 2: Functions and Their Graphs

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47. \( f(x) = -5x + 4 \)
   Domain: \( \{ x \mid x \text{ is any real number} \} \)

49. \( f(x) = \frac{x}{x^2 + 1} \)
   Domain: \( \{ x \mid x \text{ is any real number} \} \)

51. \( g(x) = \frac{x}{x^2 - 16} \)
   \( x^2 - 16 \neq 0 \)
   \( x^2 - 16 \Rightarrow x \neq \pm 4 \)
   Domain: \( \{ x \mid x \neq -4, x \neq 4 \} \)

53. \( F(x) = \frac{x - 2}{x^2 + x} \)
   \( x^2 + x \neq 0 \)
   \( x(x^2 + 1) \neq 0 \)
   \( x \neq 0, \ x^2 
eq -1 \)
   Domain: \( \{ x \mid x \neq 0 \} \)

55. \( h(x) = \sqrt{3x - 12} \)
   \( 3x - 12 \geq 0 \)
   \( 3x \geq 12 \)
   \( x \geq 4 \)
   Domain: \( \{ x \mid x \geq 4 \} \)

57. \( f(x) = \frac{4}{\sqrt{x - 9}} \)
   \( x - 9 > 0 \)
   \( x > 9 \)
   Domain: \( \{ x \mid x > 9 \} \)

59. \( p(x) = \frac{-2}{\sqrt{x - 1}} = \frac{\sqrt{5}}{\sqrt{x - 1}} \)
   \( x - 1 > 0 \)
   \( x > 1 \)
   Domain: \( \{ x \mid x > 1 \} \)

61. \( f(x) = 3x + 4 \quad g(x) = 2x - 3 \)
   a. \( (f + g)(x) = 3x + 4 + 2x - 3 = 5x + 1 \)
      The domain is \( \{ x \mid x \text{ is any real number} \} \).
   b. \( (f - g)(x) = (3x + 4) - (2x - 3) = 3x + 4 - 2x + 3 = x + 7 \)
      The domain is \( \{ x \mid x \text{ is any real number} \} \).
   c. \( (f \cdot g)(x) = (3x + 4)(2x - 3) = 6x^2 - 9x + 8x - 12 = 6x^2 - x - 12 \)
      The domain is \( \{ x \mid x \text{ is any real number} \} \).
   d. \( \left( \frac{f}{g} \right)(x) = \frac{3x + 4}{2x - 3} \)
      \( 2x - 3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow x \neq \frac{3}{2} \)
      The domain is \( \{ x \mid x \neq \frac{3}{2} \} \).

63. \( f(x) = x - 1 \quad g(x) = 2x^2 \)
   a. \( (f + g)(x) = x - 1 + 2x^2 = 2x^2 + x - 1 \)
      The domain is \( \{ x \mid x \text{ is any real number} \} \).
   b. \( (f - g)(x) = (x - 1) - (2x^2) = x - 1 - 2x^2 = -2x^2 + x - 1 \)
      The domain is \( \{ x \mid x \text{ is any real number} \} \)
c. \((f \cdot g)(x) = (x-1)(2x^2) = 2x^3 - 2x^2\)
   The domain is \(\{x \mid x \text{ is any real number}\}\).

d. \(\left(\frac{f}{g}\right)(x) = \frac{x-1}{2x^2}\)
   The domain is \(\{x \mid x \neq 0\}\).

65. \(f(x) = \sqrt{x}\) \(g(x) = 3x - 5\)

a. \((f + g)(x) = \sqrt{x} + 3x - 5\)
   The domain is \(\{x \mid x \geq 0\}\).

b. \((f - g)(x) = \sqrt{x} - (3x - 5) = \sqrt{x} - 3x + 5\)
   The domain is \(\{x \mid x \geq 0\}\).

c. \((f \cdot g)(x) = \sqrt{x}(3x - 5) = 3x\sqrt{x} - 5\sqrt{x}\)
   The domain is \(\{x \mid x \geq 0\}\).

d. \(\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{3x - 5}\)
   \(x \geq 0\) and \(3x - 5 \neq 0\)
   \(3x - 5 \Rightarrow x \neq \frac{5}{3}\)
   The domain is \(\{x \mid x \geq 0 \text{ and } x \neq \frac{5}{3}\}\).

67. \(f(x) = 1 + \frac{1}{x}\) \(g(x) = \frac{1}{x}\)

a. \((f + g)(x) = 1 + \frac{1}{x} + \frac{1}{x} = 1 + \frac{2}{x}\)
   The domain is \(\{x \mid x \neq 0\}\).

b. \((f - g)(x) = 1 + \frac{1}{x} - \frac{1}{x} = 1\)
   The domain is \(\{x \mid x \neq 0\}\).

c. \((f \cdot g)(x) = \left(1 + \frac{1}{x}\right) \cdot \frac{1}{x} = \frac{1}{x} + \frac{1}{x^2}\)
   The domain is \(\{x \mid x \neq 0\}\).

d. \(\left(\frac{f}{g}\right)(x) = \frac{1 + \frac{1}{x}}{x} = \frac{x + 1}{x^2} = \frac{x + 1}{x} \cdot \frac{1}{x} = x + 1\)
   The domain is \(\{x \mid x \neq 0\}\).

69. \(f(x) = \frac{2x + 3}{3x - 2}\) \(g(x) = \frac{4x}{3x - 2}\)

a. \((f + g)(x) = \frac{2x + 3}{3x - 2} + \frac{4x}{3x - 2}\)
   \(= \frac{2x + 3 + 4x}{3x - 2}\)
   \(= \frac{6x + 3}{3x - 2}\)
   \(3x - 2 \neq 0\)
   \(3x / 2 \Rightarrow x \neq \frac{2}{3}\)
   The domain is \(\{x \mid x \neq \frac{2}{3}\}\).

b. \((f - g)(x) = \frac{2x + 3}{3x - 2} - \frac{4x}{3x - 2}\)
   \(= \frac{2x + 3 - 4x}{3x - 2}\)
   \(= \frac{-2x + 3}{3x - 2}\)
   \(3x - 2 \neq 0\)
   \(3x / 2 \Rightarrow x \neq \frac{2}{3}\)
   The domain is \(\{x \mid x \neq \frac{2}{3}\}\).

c. \((f \cdot g)(x) = \left(\frac{2x + 3}{3x - 2}\right) \left(\frac{4x}{3x - 2}\right)\)
   \(= \frac{8x^2 + 12x}{(3x - 2)^2}\)
   \(3x - 2 \neq 0\)
   \(3x / 2 \Rightarrow x \neq \frac{2}{3}\)
   The domain is \(\{x \mid x \neq \frac{2}{3}\}\).

d. \(\left(\frac{f}{g}\right)(x) = \frac{2x + 3}{3x - 2} \cdot \frac{3x - 2}{4x}\)
   \(= \frac{2x + 3}{4x}\)
   \(3x - 2 \neq 0\) and \(x \neq 0\)
   \(3x \neq 2\) \(\Rightarrow x \neq \frac{2}{3}\)
   The domain is \(\{x \mid x \neq \frac{2}{3} \text{ and } x \neq 0\}\).
71. \( f(x) = 3x + 1 \quad (f + g)(x) = 6 - \frac{1}{2}x \)

\[
6 - \frac{1}{2}x = 3x + 1 \quad \text{g}(x)
\]

\[
5 - \frac{7}{2} = \text{g}(x)
\]

\[
g(x) = 5 - \frac{7}{2}x
\]

73. \( f(x) = 4x + 3 \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) + 3 - 4x - 3}{h}
\]

\[
= \frac{4x + 4h + 3 - 4x - 3}{h}
\]

\[
= \frac{4h}{h} = 4
\]

75. \( f(x) = x^2 - x + 4 \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x+h) + 4 - (x^2 - x + 4)}{h}
\]

\[
= \frac{x^2 + 2xh + h^2 - x - h + 4 - x^2 + x - 4}{h}
\]

\[
= \frac{2xh + h^2 - h}{h}
\]

\[
= 2x + h - 1
\]

77. \( f(x) = x^3 - 2 \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 2 - (x^3 - 2)}{h}
\]

\[
= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2 - x^3 + 2}{h}
\]

\[
= \frac{3x^2h + 3xh^2 + h^3}{h}
\]

\[
= 3x^2 + 3xh + h^2
\]

79. \( f(x) = 2x^3 + Ax^2 + 4x - 5 \) and \( f(2) = 5 \)

\[
f(2) = 2(2)^3 + A(2)^2 + 4(2) - 5
\]

\[
5 = 16 + 4A + 8 - 5
\]

\[
5 = 4A + 19
\]

\[
-14 = 4A
\]

\[
A = -\frac{7}{2}
\]

81. \( f(x) = \frac{3x + 8}{2x - A} \) and \( f(0) = 2 \)

\[
f(0) = \frac{3(0) + 8}{2(0) - A}
\]

\[
2 = \frac{8}{-A}
\]

\[
-2A = 8
\]

\[
A = -4
\]

83. \( f(x) = \frac{2x - A}{x - 3} \) and \( f(4) = 0 \)

\[
f(4) = \frac{2(4) - A}{4 - 3}
\]

\[
0 = \frac{8 - A}{1}
\]

\[
0 = 8 - A
\]

\[
A = 8
\]

\[
f \text{ is undefined when } x = 3.
\]

85. Let \( x \) represent the length of the rectangle.

Then, \( \frac{x}{2} \) represents the width of the rectangle since the length is twice the width.

The function for the area is:

\[
A(x) = x \cdot \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2
\]

87. Let \( x \) represent the number of hours worked.

The function for the gross salary is: \( G(x) = 10x \)
89. a. \( H(1) = 20 - 4.9(1)^2 \\
= 20 - 4.9 \\
= 15.1 \text{ meters} \)
\( H(1.1) = 20 - 4.9(1.1)^2 = 20 - 4.9(1.21) \\
= 20 - 5.929 \\
= 14.071 \text{ meters} \)
\( H(1.2) = 20 - 4.9(1.2)^2 \\
= 20 - 4.9(1.44) \\
= 20 - 7.056 \\
= 12.944 \text{ meters} \)
\( H(1.3) = 20 - 4.9(1.3)^2 \\
= 20 - 4.9(1.69) \\
= 20 - 8.281 \\
= 11.719 \text{ meters} \)

b. \( H(x) = 15 : \\
15 = 20 - 4.9x^2 \\
-5 = -4.9x^2 \\
x^2 \approx 1.0204 \\
x \approx 1.01 \text{ seconds} \)
\( H(x) = 10 : \\
10 = 20 - 4.9x^2 \\
-10 = -4.9x^2 \\
x^2 \approx 2.0408 \\
x \approx 1.43 \text{ seconds} \)
\( H(x) = 5 : \\
5 = 20 - 4.9x^2 \\
-15 = -4.9x^2 \\
x^2 \approx 3.0612 \\
x \approx 1.75 \text{ seconds} \)

c. \( H(x) = 0 \\
0 = 20 - 4.9x^2 \\
-20 = -4.9x^2 \\
x^2 \approx 4.0816 \\
x \approx 2.02 \text{ seconds} \)

b. \( H(x) = 15 : \\
15 = 20 - 4.9x^2 \\
-5 = -4.9x^2 \\
x^2 \approx 1.0204 \\
x \approx 1.01 \text{ seconds} \)
\( H(x) = 10 : \\
10 = 20 - 4.9x^2 \\
-10 = -4.9x^2 \\
x^2 \approx 2.0408 \\
x \approx 1.43 \text{ seconds} \)
\( H(x) = 5 : \\
5 = 20 - 4.9x^2 \\
-15 = -4.9x^2 \\
x^2 \approx 3.0612 \\
x \approx 1.75 \text{ seconds} \)

c. \( H(x) = 0 \\
0 = 20 - 4.9x^2 \\
-20 = -4.9x^2 \\
x^2 \approx 4.0816 \\
x \approx 2.02 \text{ seconds} \)

91. \( C(x) = 100 + \frac{x + \frac{36,000}{x}}{10} \)

a. \( C(500) = 100 + \frac{500 + \frac{36,000}{500}}{10} = 100 + 50 + 72 = \$222 \)

b. \( C(450) = 100 + \frac{450 + \frac{36,000}{450}}{10} = 100 + 45 + 80 = \$225 \)

c. \( C(600) = 100 + \frac{600 + \frac{36,000}{600}}{10} = 100 + 60 + 60 = \$220 \)

d. \( C(400) = 100 + \frac{400 + \frac{36,000}{400}}{10} = 100 + 40 + 90 = \$230 \)

93. \( R(x) = \left( \frac{L}{P} \right) (x) = \frac{L(x)}{P(x)} \)

95. \( H(x) = (P - I)(x) = P(x) - I(x) \)

97. a. \( h(x) = 2x \\
h(a + b) = 2(a + b) = 2a + 2b \\
= h(a) + h(b) \\
h(x) = 2x \text{ has the property.} \)

b. \( g(x) = x^2 \\
g(a + b) = (a + b)^2 = a^2 + 2ab + b^2 \\
Since \\
\[ a^2 + 2ab + b^2 \neq a^2 + b^2 = g(a) + g(b), \]
g(x) = x^2 does not have the property.

c. \( F(x) = 5x - 2 \\
F(a + b) = 5(a + b) - 2 = 5a + 5b - 2 \\
Since \\
\[ 5a + 5b - 2 \neq 5a - 2 + 5b - 2 = F(a) + F(b), \]
\( F(x) = 5x - 2 \text{ does not have the property.} \)
d. \( G(x) = \frac{1}{x} \)

\[ G(a + b) = \frac{1}{a + b} \neq \frac{1}{a} + \frac{1}{b} = G(a) + G(b) \]

\( G(x) = \frac{1}{x} \) does not have the property.

99. Answers will vary.

Section 2.2

1. \( x^2 + 4y^2 = 16 \)

x-intercepts:

\[ x^2 + 4(0)^2 = 16 \]

\[ x^2 = 16 \]

\[ x = \pm 4 \implies (-4,0),(4,0) \]

y-intercepts:

\[ (0)^2 + 4y^2 = 16 \]

\[ 4y^2 = 16 \]

\[ y^2 = 4 \]

\[ y = \pm 2 \implies (0,-2),(0,2) \]

3. vertical

5. \( f(x) = ax^2 + 4 \)

\[ a(-1)^2 + 4 = 2 \implies a = -2 \]

7. False; e.g. \( y = \frac{1}{x} \).

9. a. \( f(0) = 3 \) since \((0,3)\) is on the graph.

\[ f(-6) = -3 \] since \((-6,-3)\) is on the graph.

b. \( f(6) = 0 \) since \((6, 0)\) is on the graph.

\[ f(11) = 1 \] since \((11, 1)\) is on the graph.

c. \( f(3) \) is positive since \( f(3) \approx 3.7 \).

d. \( f(-4) \) is negative since \( f(-4) \approx -1 \).

e. \( f(x) = 0 \) when \( x = -3, x = 6, \) and \( x = 10 \).

f. \( f(x) > 0 \) when \(-3 < x < 6,\) and \(10 < x \leq 11 \).

g. The domain of \( f \) is

\[ \{x \mid -6 \leq x \leq 11\} \quad \text{or} \quad [-6,11] \]

h. The range of \( f \) is

\[ \{y \mid -3 \leq y \leq 4\} \quad \text{or} \quad [-3,4] \].

i. The x-intercepts are \((-3, 0), (6, 0),\) and \((10, 0)\).

j. The y-intercept is \((0, 3)\).

k. The line \( y = \frac{1}{2} \) intersects the graph 3 times.

l. The line \( x = 5 \) intersects the graph 1 time.

m. \( f(x) = 3 \) when \( x = 0 \) and \( x = 4 \).

n. \( f(x) = -2 \) when \( x = -5 \) and \( x = 8 \).

11. Not a function since vertical lines will intersect the graph in more than one point.

13. Function

a. Domain: \( \{x \mid -\pi \leq x \leq \pi\} \)

Range: \( \{y \mid -1 \leq y \leq 1\} \)

b. Intercepts: \( \left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0,1) \)

c. Symmetry about y-axis.

15. Not a function since vertical lines will intersect the graph in more than one point.

17. Function

a. Domain: \( \{x \mid x > 0\} \)

Range: \( \{y \mid y \text{ is any real number}\} \)

b. Intercepts: \((1, 0)\)

c. None

19. Function

a. Domain: \( \{x \mid x \text{ is any real number}\} \)

Range: \( \{y \mid y \leq 2\} \)

b. Intercepts: \((-3, 0), (3, 0), (0,2)\)

c. Symmetry about y-axis.

21. Function

a. Domain: \( \{x \mid x \text{ is any real number}\} \)

Range: \( \{y \mid y \geq -3\} \)

b. Intercepts: \((1, 0), (3,0), (0,9)\)

c. None
23. \( f(x) = 2x^2 - x - 1 \)
   a. \( f(-1) = 2(-1)^2 - (-1) - 1 = 2 \)
      The point \((-1, 2)\) is on the graph of \(f\).
   b. \( f(-2) = 2(-2)^2 - (-2) - 1 = 9 \)
      The point \((-2, 9)\) is on the graph of \(f\).
   c. Solve for \(x\):
      
      \[
      -1 = 2x^2 - x - 1 \\
      0 = 2x^2 - x \\
      0 = x(2x - 1) 
      \Rightarrow x = 0, x = \frac{1}{2} \\
      \]
      \((-1, 0)\) and \(\left(\frac{1}{2}, -1\right)\) are on the graph of \(f\).
   d. The domain of \(f\) is \(\{x \mid x \text{ is any real number}\}\).
   e. \(y\)-intercept:
      
      \[
      f(0) = 2(0)^2 - 0 - 1 = -1 \Rightarrow (0, -1) 
      \]

25. \( f(x) = \frac{x+2}{x-6} \)
   a. \( f(3) = \frac{3+2}{3-6} = \frac{5}{-3} = -\frac{5}{3} \neq 14 \)
      The point \((3, 14)\) is not on the graph of \(f\).
   b. \( f(4) = \frac{4+2}{4-6} = \frac{6}{-2} = -3 \)
      The point \((4, -3)\) is on the graph of \(f\).
   c. Solve for \(x\):
      
      \[
      2 = \frac{x+2}{x-6} \\
      2(x-6) = x+2 \\
      2x - 12 = x + 2 \\
      x = 14 \\
      \]
      \((14, 2)\) is a point on the graph of \(f\).
   d. The domain of \(f\) is \(\{x \mid x \neq 6\}\).
29. \( h(x) = \frac{-32x^2}{130^2} + x \)

a. \( h(100) = \frac{-32(100)^2}{130^2} + 100 \)
\( = \frac{-320,000}{16,900} + 100 \approx 81.07 \) feet

b. \( h(300) = \frac{-32(300)^2}{130^2} + 300 \)
\( = \frac{-2,880,000}{16,900} + 300 \approx 129.59 \) feet

c. \( h(500) = \frac{-32(500)^2}{130^2} + 500 \)
\( = \frac{-8,000,000}{16,900} + 500 \approx 26.63 \) feet

d. Solving \( h(x) = \frac{-32x^2}{130^2} + x = 0 \)
\( \frac{-32x^2}{130^2} + x = 0 \)
\( x \left( \frac{-32x + 1}{130^2} \right) = 0 \)
\( x = 0 \) or \( \frac{-32x}{130^2} + 1 = 0 \)
\( 1 = \frac{32x}{130^2} \)
\( 130^2 = 32x \)
\( x = \frac{130^2}{32} = 528.125 \) feet
Therefore, the golf ball travels 528.125 feet.

e. \( y_1 = \frac{-32x^2}{130^2} + x \)

f. Use INTERSECT on the graphs of
\( y_1 = \frac{-32x^2}{130^2} + x \) and \( y_2 = 90 \).

The ball reaches a height of 90 feet twice.
The first time is when the ball has traveled approximately 115 feet, and the second time is when the ball has traveled about 413 feet.

g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.

h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.
31. \( C(x) = 100 + \frac{x}{10} + \frac{36000}{x} \)

a. Graphing:

b. TblStart = 0; ΔTbl = 50

<table>
<thead>
<tr>
<th>x</th>
<th>y1</th>
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<tbody>
<tr>
<td>50</td>
<td>14</td>
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<td>100</td>
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<td>580</td>
<td>60</td>
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<td>600</td>
<td>60</td>
</tr>
</tbody>
</table>

32. The cost per passenger is minimized to about $220 when the ground speed is roughly 600 miles per hour.

33. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the y-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

35. The graph of a function can have at most one y-intercept.

37. (a) III; (b) IV; (c) I; (d) V; (e) II

39. The equation has symmetry with respect to the y-axis only.
5. \( y = x^2 - 9 \)
   x-intercepts:
   
   \[ \begin{align*}
   0 &= x^2 - 9 \\
   x^2 &= 9 \\
   x &= \pm 3
   \end{align*} \]
   
   y-intercept:
   
   \[ y = (0)^2 - 9 = -9 \]
   
   The intercepts are \((-3, 0), (3, 0), \text{ and } (0, -9)\).
   
7. even; odd
   
9. True
   
11. Yes
   
13. No, it only increases on \((5, 10)\).
   
15. \( f \) is increasing on the intervals \((-8, -2), (0, 2), (5, \infty)\).
   
17. Yes. The local maximum at \( x = 2 \) is 10.
   
19. \( f \) has local maxima at \( x = -2 \) and \( x = 2 \). The local maxima are 6 and 10, respectively.
   
21. a. Intercepts: \((-2, 0), (2, 0), \text{ and } (0, 3)\).
   b. Domain: \( \{x \mid -4 \leq x \leq 4\} \);
      Range: \( \{y \mid 0 \leq y \leq 3\} \).
   c. Increasing: \((-2, 0) \text{ and } (2, 4)\);
      Decreasing: \((-4, -2) \text{ and } (0, 2)\).
   d. Since the graph is symmetric with respect to the \( y \)-axis, the function is even.
   
23. a. Intercepts: \((0, 1)\).
   b. Domain: \( \{x \mid x \text{ is any real number}\} \);
      Range: \( \{y \mid y > 0\} \).
   c. Increasing: \((-\infty, \infty)\); Decreasing: never.
   d. Since the graph is not symmetric with respect to the \( y \)-axis or the origin, the function is neither even nor odd.
   
25. a. Intercepts: \((-\pi, 0), (\pi, 0), \text{ and } (0, 0)\).
   b. Domain: \( \{x \mid -\pi \leq x \leq \pi\} \);
      Range: \( \{y \mid -1 \leq y \leq 1\} \).
   c. Increasing: \( \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\);
      Decreasing: \( (-\pi, -\frac{\pi}{2}) \text{ and } \left(\frac{\pi}{2}, \pi\right)\).
   d. Since the graph is symmetric with respect to the origin, the function is odd.
   
27. a. Intercepts: \( \left(\frac{1}{2}, 0\right), \left(\frac{5}{2}, 0\right), \text{ and } \left(0, \frac{1}{2}\right)\).
   b. Domain: \( \{x \mid -3 \leq x \leq 3\} \);
      Range: \( \{y \mid -1 \leq y \leq 2\} \).
   c. Increasing: \((2, 3)\); Decreasing: \((-1, 1)\);
      Constant: \((-3, -1) \text{ and } (1, 2\).
   d. Since the graph is not symmetric with respect to the \( y \)-axis or the origin, the function is neither even nor odd.
   
29. a. \( f \) has a local maximum of 3 at \( x = 0 \).
   b. \( f \) has a local minimum of 0 at both \( x = -2 \) and \( x = 2 \).
   
31. a. \( f \) has a local maximum of 1 at \( x = \frac{\pi}{2} \).
   b. \( f \) has a local minimum of \(-1 \) at \( x = -\frac{\pi}{2} \).
   
33. \( f(x) = 4x^3 \)
      \[ f(-x) = 4(-x)^3 = -4x^3 = -f(x) \]
      Therefore, \( f \) is odd.
   
35. \( g(x) = -3x^2 - 5 \)
      \[ g(-x) = -3(-x)^2 - 5 = -3x^2 - 5 = g(x) \]
      Therefore, \( g \) is even.
   
37. \( F(x) = \sqrt{x} \)
      \[ F(-x) = \sqrt{-x} = -\sqrt{x} = -F(x) \]
      Therefore, \( F \) is odd.
   
39. \( f(x) = x + |x| \)
      \[ f(-x) = -x + |-x| = -x + |x| \]
      \( f \) is neither even nor odd.
41. \( g(x) = \frac{1}{x^2} \)  
\[ g(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = g(x) \]  
Therefore, \( g \) is even.

43. \( h(x) = \frac{-x^3}{3x^2 - 9} \)  
\[ h(-x) = \frac{(-(-x))^3}{3((-x)^2) - 9} = \frac{x^3}{3x^2 - 9} = -h(x) \]  
Therefore, \( h \) is odd.

45. \( f(x) = x^3 - 3x + 2 \) on the interval \((-2, 2)\)  
Use MAXIMUM and MINIMUM on the graph of \( y_1 = x^3 - 3x + 2 \).

47. \( f(x) = x^5 - x^3 \) on the interval \((-2, 2)\)  
Use MAXIMUM and MINIMUM on the graph of \( y_1 = x^5 - x^3 \).
Chapter 2: Functions and Their Graphs

51. \( f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3 \) on the interval \((-3, 2)\)

Use MAXIMUM and MINIMUM on the graph of \( y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3 \).

\[
\begin{align*}
\text{Local maximum at:} & \quad (0, 3) \\
\text{Local minimum at:} & \quad (-1.87, 0.95), (0.97, 2.65) \\
\text{\( f \) is increasing on:} & \quad (-1.87, 0) \text{ and } (0.97, 2) \\
\text{\( f \) is decreasing on:} & \quad (-3, -1.87) \text{ and } (0, 0.97)
\end{align*}
\]

53. \( f(x) = -2x^2 + 4 \)

a. Average rate of change of \( f \) from\( x = 0 \) to \( x = 2 \):

\[
\frac{f(2) - f(0)}{2 - 0} = \frac{(-2(2)^2 + 4) - (-2(0)^2 + 4)}{2} = \frac{-8 - 4}{2} = -6
\]

b. Average rate of change of \( f \) from \( x = 1 \) to \( x = 3 \):

\[
\frac{f(3) - f(1)}{3 - 1} = \frac{(-2(3)^2 + 4) - (-2(1)^2 + 4)}{2} = \frac{-16 - 4}{2} = -10
\]

c. Average rate of change of \( f \) from \( x = 1 \) to \( x = 4 \):

\[
\frac{f(4) - f(1)}{4 - 1} = \frac{(-2(4)^2 + 4) - (-2(1)^2 + 4)}{3} = \frac{-30}{3} = -10
\]

55. \( g(x) = x^3 - 2x + 1 \)

a. Average rate of change of \( g \) from \( x = -3 \) to \( x = -2 \):

\[
\frac{g(-2) - g(-3)}{-2 - (-3)} = \frac{(-2)^3 - 2(-2) + 1 - (-3)^3 - 2(-3) + 1}{1} = \frac{(-3) - (-20)}{1} = \frac{17}{1} = 17
\]

b. Average rate of change of \( g \) from \( x = -1 \) to \( x = 1 \):

\[
\frac{g(1) - g(-1)}{1 - (-1)} = \frac{(1)^3 - 2(1) + 1 - (-1)^3 - 2(-1) + 1}{2} = \frac{0 - 2}{2} = -1
\]

c. Average rate of change of \( g \) from \( x = 1 \) to \( x = 3 \):

\[
\frac{g(3) - g(1)}{3 - 1} = \frac{(3)^3 - 2(3) + 1 - (1)^3 - 2(1) + 1}{2} = \frac{2}{2} = 11
\]
57. \( f(x) = 5x - 2 \)
   a. Average rate of change of \( f \) from 1 to \( x \):
   \[
   \frac{f(x) - f(1)}{x - 1} = \frac{(5x - 2) - (5(1) - 2)}{x - 1} = \frac{5x - 2 - 3}{x - 1} = \frac{5x - 5}{x - 1} = \frac{5(x - 1)}{x - 1} = 5
   \]
   b. The average rate of change of \( f \) from 1 to \( x \) is a constant 5. Therefore, the average rate of change of \( f \) from 1 to 3 is 5. The slope of the secant line joining \((1, f(1))\) and \((3, f(3))\) is 5.
   c. We use the point-slope form to find the equation of the secant line:
   \[
   y - y_1 = m_{sec}(x - x_1)
   \]
   \[
   y - 2 = -1(x - (-2))
   \]
   \[
   y - 2 = -x + 2 
   \]
   \[
   y = -x
   \]
   d. The graph below shows the graph of \( f \) along with the secant line \( y = -x \).

59. \( g(x) = x^2 - 2 \)
   a. Average rate of change of \( g \) from \(-2\) to \( x \):
   \[
   \frac{g(x) - g(-2)}{x - (-2)} = \frac{[x^2 - 2] - [(-2)^2 - 2]}{x + 2} = \frac{(x^2 - 2) - (2)}{x + 2} = \frac{x^2 - 4}{x + 2} = \frac{(x + 2)(x - 2)}{x + 2} = x - 2
   \]
   b. The average rate of change of \( g \) from \(-2\) to \( x \) is given by \( x - 2 \). Therefore, the average rate of change of \( g \) from \(-2\) to 1 is \(-1 - 2 = -1\). The slope of the secant line joining \((-2, g(-2))\) and \((1, g(1))\) is \(-1\).

61. \( h(x) = x^2 - 2x \)
   a. Average rate of change of \( h \) from 2 to \( x \):
   \[
   \frac{h(x) - h(2)}{x - 2} = \frac{[x^2 - 2x] - [(2)^2 - 2(2)]}{x - 2} = \frac{x^2 - 2x - (0)}{x - 2} = \frac{x^2 - 2x}{x - 2} = \frac{x(x - 2)}{x - 2} = x
   \]
   b. The average rate of change of \( h \) from 2 to \( x \) is given by \( x \). Therefore, the average rate of change of \( h \) from 2 to 4 is 4. The slope of the secant line joining \((2, h(2))\) and \((4, h(4))\) is 4.
   c. We use the point-slope form to find the equation of the secant line:
   \[
   y - y_1 = m_{sec}(x - x_1)
   \]
   \[
   y - 4 = 4(x - 2)
   \]
   \[
   y = 4x - 8
   \]
   d. The graph below shows the graph of \( h \) along with the secant line \( y = 4x - 8 \).
63. a. length = 24 - 2x; width = 24 - 2x; height = x
   
   \[ V(x) = x(24 - 2x)(24 - 2x) = x(24 - 2x)^2 \]

   b. \[ V(3) = 3(24 - 2(3))^2 = 3(18)^2 = 3(324) = 972 \text{ cu.in.} \]

   c. \[ V(10) = 10(24 - 2(10))^2 = 10(4)^2 = 10(16) = 160 \text{ cu.in.} \]

   d. \[ y_1 = x(24 - 2x)^2 \]

65. a. \[ y_1 = -16x^2 + 80x + 6 \]

   b. Use MAXIMUM. The maximum height occurs when \( t = 2.5 \) seconds.

   c. From the graph, the maximum height is 106 feet.

67. \( C(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x} \)

   a. \[ y_1 = 0.3x^2 + 21x - 251 + \frac{2500}{x} \]

   b. Use MINIMUM. The average cost is minimized when approximately 9.66 lawnmowers are produced per hour.

   c. The minimum average cost is approximately $238.65.

69. (a), (b), (e)

   c. Average rate of change = \( \frac{28000 - 0}{25 - 0} = \frac{28000}{25} = 1120 \) dollars/bicycle

   d. For each additional bicycle sold between 0 and 25, the total revenue increases by (an average of) $1120.

   f. Average rate of change = \( \frac{64835 - 62360}{223 - 190} = \frac{2475}{33} = 75 \) dollars per bicycle
g. For each additional bicycle sold between 190 and 223, the total revenue increases by (an average of) $75.

h. The average rate of change of revenue is decreasing as the number of bicycles increases.

71. \( f(x) = x^2 \)

a. Average rate of change of \( f \) from \( x = 0 \) to \( x = 1 \):
\[
\frac{f(1) - f(0)}{1-0} = \frac{1^2 - 0^2}{1} = 1
\]

b. Average rate of change of \( f \) from \( x = 0 \) to \( x = 0.5 \):
\[
\frac{f(0.5) - f(0)}{0.5-0} = \frac{(0.5)^2 - 0^2}{0.5} = \frac{0.25}{0.5} = 0.5
\]

c. Average rate of change of \( f \) from \( x = 0 \) to \( x = 0.1 \):
\[
\frac{f(0.1) - f(0)}{0.1-0} = \frac{(0.1)^2 - 0^2}{0.1} = \frac{0.01}{0.1} = 0.1
\]

d. Average rate of change of \( f \) from \( x = 0 \) to \( x = 0.01 \):
\[
\frac{f(0.01) - f(0)}{0.01-0} = \frac{(0.01)^2 - 0^2}{0.01} = \frac{0.0001}{0.01} = 0.01
\]

e. Average rate of change of \( f \) from \( x = 0 \) to \( x = 0.001 \):
\[
\frac{f(0.001) - f(0)}{0.001-0} = \frac{(0.001)^2 - 0^2}{0.001} = \frac{0.000001}{0.001} = 0.001
\]

f. Graphing the secant lines:

g. The secant lines are beginning to look more and more like the tangent line to the graph of \( f \) at the point where \( x = 0 \).

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.
Chapter 2: Functions and Their Graphs

73. \( f(x) = 2x + 5 \)

a. \( m_{sec} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 5 - 2x - 5}{h} = \frac{2h}{h} = 2 \)

b. When \( x = 1 \):
   - \( h = 0.5 \Rightarrow m_{sec} = 2 \)
   - \( h = 0.1 \Rightarrow m_{sec} = 2 \)
   - \( h = 0.01 \Rightarrow m_{sec} = 2 \)
   - As \( h \to 0 \), \( m_{sec} \to 2 \)

c. Using the point \( (1, f(1)) = (1, 7) \) and slope, \( m = 2 \), we get the secant line:
   \[
   y - 7 = 2(x - 1)
   \]
   \[
   y = 2x + 5
   \]

d. Graphing:

The graph and the secant line coincide.

75. \( f(x) = x^2 + 2x \)

a. \( m_{sec} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2 \)

b. When \( x = 1 \),
   - \( h = 0.5 \Rightarrow m_{sec} = 2 \cdot 1 + 0.5 + 2 = 4.5 \)
   - \( h = 0.1 \Rightarrow m_{sec} = 2 \cdot 1 + 0.1 + 2 = 4.1 \)
   - \( h = 0.01 \Rightarrow m_{sec} = 2 \cdot 1 + 0.01 + 2 = 4.01 \)
   - As \( h \to 0 \), \( m_{sec} \to 2 \cdot 1 + 0 + 2 = 4 \)

c. Using point \( (1, f(1)) = (1, 3) \) and slope \( = 4.01 \), we get the secant line:
   \[
   y - 3 = 4.01(x - 1)
   \]
   \[
   y = 4.01x - 4.01
   \]
   \[
   y = 4.01x - 1.01
   \]

d. Graphing:

77. \( f(x) = 2x^2 - 3x + 1 \)

a. \( m_{sec} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} = \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} = \frac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3 \)

b. When \( x = 1 \),
   - \( h = 0.5 \Rightarrow m_{sec} = 4 \cdot 1 + 2 \cdot (0.5) - 3 = 2 \)
   - \( h = 0.1 \Rightarrow m_{sec} = 4 \cdot 1 + 2 \cdot (0.1) - 3 = 1.2 \)
   - \( h = 0.01 \Rightarrow m_{sec} = 4 \cdot 1 + 2 \cdot (0.01) - 3 = 1.02 \)
   - As \( h \to 0 \), \( m_{sec} \to 4 \cdot 1 + 2 \cdot (0) - 3 = 1 \)

c. Using point \( (1, f(1)) = (1, 0) \) and slope \( = 1.02 \), we get the secant line:
   \[
   y - 0 = 1.02(x - 1)
   \]
   \[
   y = 1.02x - 1.02
   \]

54
d. Graphing:

\[
\begin{array}{c}
\text{f(x)} = \frac{1}{x} \\
\end{array}
\]

79. \( f(x) = \frac{1}{x} \)

a. \( m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{\left( \frac{1}{x+h} - \frac{1}{x} \right)}{h} \)

\[
= \frac{\left( x - (x+h) \right)}{(x+h)x} \cdot \frac{1}{h} = \frac{1}{(x+h)x} \cdot \frac{1}{h}
\]

b. When \( x = 1 \),

\( h = 0.5 \Rightarrow m_{\text{sec}} = \frac{-1}{(1+0.5)(1)} \approx -0.667 \)

\( h = 0.1 \Rightarrow m_{\text{sec}} = \frac{-1}{(1+0.1)(1)} \approx -0.909 \)

\( h = 0.01 \Rightarrow m_{\text{sec}} = \frac{-1}{(1+0.01)(1)} \approx -0.990 \)

as \( h \to 0, \quad m_{\text{sec}} \to \frac{-1}{(1+0)(1)} = \frac{-1}{1} = -1 \)

c. Using point \((1, f(1)) = (1,1)\) and slope = 0.990, we get the secant line:

\[
\begin{align*}
y - 1 & = 0.99(x - 1) \\
y - 1 & = 0.99x + 0.99 \\
y & = 0.99x + 1.99
\end{align*}
\]

81. Answers will vary. One possibility follows:

83. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of \( f \) could not "turn" and cross it again or it would start to decrease.

85. To be an even function we need \( f(-x) = f(x) \) and to be an odd function we need \( f(-x) = -f(x) \). In order for a function to be both even and odd, we would need \( f(x) = -f(x) \). This is only possible if \( f(x) = 0 \).
Section 2.4

1. From the equation \( y = 2x - 3 \), we see that the \( y \)-intercept is \(-3\). Thus, the point \((0, -3)\) is on the graph. We can obtain a second point by choosing a value for \( x \) and finding the corresponding value for \( y \).
   Let \( x = 2 \), then \( y = 2(2) - 3 = 1 \). Thus, the point \((2, 1)\) is also on the graph. Plotting the two points and connecting with a line yields the graph below.

3. We can use the point-slope form of a line to obtain the equation.
   \[
   y - y_1 = m(x - x_1)
   \]
   \[
   y - 5 = -3(x - (-1))
   \]
   \[
   y - 5 = -3(x + 1)
   \]
   \[
   y - 5 = -3x - 3
   \]
   \[
   y = -3x + 2
   \]

5. slope; \( y \)-intercept

7. \( y = kx \)

9. True

11. \( f(x) = 2x + 3 \)
   Slope = average rate of change = 2; 
   \( y \)-intercept = 3

13. \( h(x) = -3x + 4 \)
   Slope = average rate of change = \(-3\); 
   \( y \)-intercept = 4

15. \( f(x) = \frac{1}{4}x - 3 \)
   Slope = average rate of change = \( \frac{1}{4} \); 
   \( y \)-intercept = \(-3\)

17. \( F(x) = 4 \)
   Slope = average rate of change = 0; 
   \( y \)-intercept = 4

19. Linear, \( m > 0 \)

21. Linear, \( m < 0 \)

23. Nonlinear
25. a. 

![Graph of linear function](image1.png)

b. Answers will vary. We select (3, 4) and (9, 16). The slope of the line containing these points is:

\[ m = \frac{16 - 4}{9 - 3} = \frac{12}{6} = 2 \]

The equation of the line is:

\[ y - y_1 = m(x - x_1) \]

\[ y - 4 = 2(x - 3) \]

\[ y = 2x - 2 \]

c. 

![Graph of linear function](image2.png)

d. Using the LINEar REGression program, the line of best fit is:

\[ y = 2.0357x - 2.3571 \]

e. 

![Graph of linear function](image3.png)

27. a. 

![Graph of linear function](image4.png)

b. Answers will vary. We select (–2, –4) and (1, 4). The slope of the line containing these points is:

\[ m = \frac{4 - (-4)}{1 - (-2)} = \frac{8}{3} \]

The equation of the line is:

\[ y - y_1 = m(x - x_1) \]

\[ y - (-4) = \frac{8}{3}(x - (-2)) \]

\[ y + 4 = \frac{8}{3}x + \frac{16}{3} \]

\[ y = \frac{8}{3}x + \frac{4}{3} \]

c. 

![Graph of linear function](image5.png)

d. Using the LINEar REGression program, the line of best fit is:

\[ y = 2.2x + 1.2 \]

e. 

![Graph of linear function](image6.png)
b. Answers will vary. We select (–20,100) and (–15,118). The slope of the line containing these points is:
\[ m = \frac{118 - 100}{-15 - (-20)} = \frac{18}{5} = 3.6 \]
The equation of the line is:
\[ y - y_1 = m(x - x_1) \]
\[ y - 100 = 3.6(x - (-20)) \]
\[ y - 100 = 3.6x + 72 \]
\[ y = 3.6x + 172 \]

c. 
\[
\begin{array}{c|c|c|c}
\hline
\text{t} & \text{B} & \text{t} & \text{B} \\
\hline
0 & 19.25 & 10 & 585.72 \\
\hline
\end{array}
\]
The average monthly benefit in 2000 was $778.22.

b. 893.72 = 19.25\(t\) + 585.72 
308 = 19.25\(t\)
\[ 16 = t \]
The average monthly benefit will be $893.72 in 2006.

c. 1000 = 19.25\(t\) + 585.72
414.28 = 19.25\(t\)
\[ 21.52 = t \]
The average monthly benefit will exceed $1000 in 2012.

35. a. 
\[ S(p) = D(p) \]
\[ -200 + 50p = 1000 - 25p \]
\[ 75p = 1200 \]
\[ p = 16 \]
The equilibrium price is $16.
\[ S(16) = -200 + 50(16) = 600 \]
The equilibrium quantity is 600 T-shirts.

b. \[ D(p) > S(p) \]
\[ 1000 - 25p > -200 + 50p \]
\[ -75p > -1200 \]
\[ p < 16 \]
The quantity demanded will exceed the quantity supplied if \( 0 < p < 16 \).

c. If demand is higher than supply, generally the price will increase. The price will continue to increase towards the equilibrium point.

37. a. 
\[ R(x) = C(x) \]
\[ 8x = 4.5x + 17,500 \]
\[ 3.5x = 17,500 \]
\[ x = 5000 \]
The company must sell 5000 units to break even.

b. To make a profit, the company must sell more than 5000 units.
39. a. Consider the data points \((x, y)\), where \(x = \) the age in years of the computer and \(y = \) the value in dollars of the computer. So we have the points \((0, 3000)\) and \((3, 0)\). The slope formula yields:

\[
\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0 - 3000}{3 - 0} = \frac{-3000}{3} = -1000 = m
\]

\((0,3000)\) is the y-intercept, so \(b = 3000\). Therefore, the linear function is \(V(x) = mx + b = -1000x + 3000\).

b. The graph of \(V(x) = -1000x + 3000\) is shown below:

![Graph of V(x) = -1000x + 3000](image)

The computer’s value after 2 years is given by:

\[
V(2) = -1000(2) + 3000 = -2000 + 3000 = $1000
\]

c. The cost of manufacturing 14 bicycles is given by \(C(14) = 90(14) + 1800 = $3060\).

d. To determine the number of bicycles, we solve the following:

\[
3780 = 90x + 1800
\]

\[
1980 = 90x
\]

\[
x = 22
\]

The company can manufacture 22 bicycles for $3780.

41. a. Let \(x = \) the number of bicycles manufactured. We can use the cost function \(C(x) = mx + b\), with \(m = 90\) and \(b = 1800\). Therefore \(C(x) = 90x + 1800\).

b. The graph of \(C(x) = 90x + 1800\) is shown below:

![Graph of C(x) = 90x + 1800](image)

c. To find when the computer will be worth $2000, we solve the following:

\[
2000 = -1000x + 3000
\]

\[
-1000 = -1000x
\]

\[
x = 1
\]

The computer will be worth $2000 after 1 year.

d. Let \(p = \) the monthly payment and \(B = \) the amount borrowed. Consider the ordered pair \((B, p)\). We can use the points \((0, 0)\) and \((1000, 6.49)\).

Now compute the slope:

\[
slope = \frac{\Delta y}{\Delta x} = \frac{6.49 - 0}{1000 - 0} = \frac{6.49}{1000} = 0.00649
\]

Therefore we have the linear function \(p(B) = 0.00649B + 0 = 0.00649B\).

If \(B = 145000\), then

\[
p = (0.00649)(145000) = $941.05
\]
47. Let $R =$ the revenue and $g =$ the number of gallons of gasoline sold. Consider the ordered pair $(g, R)$. We can use the points $(0, 0)$ and $(12, 23.40)$. Now compute the slope:

$$\text{slope} = \frac{23.40 - 0}{12 - 0} = \frac{23.40}{12} = 1.95$$

Therefore we have the linear function $R(g) = 1.95g + 0 = 1.95g$.

If $g = 10.5$, then $R(10.5) = 1.95(10.5) = 20.48$.

49. $W = kS$

$1.875 = k(15)$

$0.125 = k$

For 40 gallons of sand:

$W = 0.125(40) = 5$ gallons of water.

51. a. [Graph]

b. [Graph]

$$C(I) = 0.9241I + 479.6584$$

c. The slope indicates that for each $1$ increase in per capita disposable income, there is an increase of $0.92$ in per capita consumption.

d. $C(28,750) = 0.9241(28,750) + 479.6584 = 27,047.53$  
When disposable income is $28,750$, the per capita consumption is about $27,048$.

e. $26,900 = 0.9241I + 479.6584$  
$26,420.3416 = 0.9241I$  
$28,590.35 = I$  
When consumption is $26,500$, the per capita disposable income is about $28,590$.

53. a. [Graph]

b. Using the LINear REGression program, the line of best fit is:

$L(G) = 0.0261G + 7.8738$

c. For each 1 day increase in Gestation period, the life expectancy increases by 0.0261 years (about 9.5 days).

d. $L(89) = 0.0261(89) + 7.8738 \approx 10.2$ years

55. a. The relation is not a function because 23 is paired with both 56 and 53.

b. [Graph]

c. Using the LINear REGression program, the line of best fit is:

$D(p) = -1.3355p + 86.1974$

d. As the price of the jeans increases by $1$, the demand for the jeans decreases by $1.3355$ pairs per day.

e. $D(p) = -1.3355p + 86.1974$

f. Domain: $\{p | 0 < p < 64\}$

Note that the $p$-intercept is roughly 64.54 and that the number of pairs cannot be negative.

g. $D(28) = -1.3355(28) + 86.1974$  
$\approx 48.8034$  
Demand is about 49 pairs.
57. The data do not follow a linear pattern so it would not make sense to find the line of best fit.

59. A linear function is odd if the y-intercept is 0. That is, if the line passes through the origin. A linear function can be even if the slope is 0.

61. A correlation coefficient of 0 implies that there is no linear relationship between the data.

Section 2.5

1. \( y = \sqrt{x} \)

3. \( y = x^3 - 8 \)
   - y-intercept:
     Let \( x = 0 \), then \( y = (0)^3 - 8 = -8 \).
   - x-intercept:
     Let \( y = 0 \), then \( 0 = x^3 - 8 \)
     \[ x^3 = 8 \]
     \[ x = 2 \]
   The intercepts are \((0, -8)\) and \((2, 0)\).

5. piecewise defined

7. False; the cube root function is odd and increasing on the interval \((-\infty, \infty)\).

9. C

11. E

13. B
23. \( f(x) = \sqrt[3]{x} \)

25. a. \( f(-2) = (-2)^2 = 4 \)
   b. \( f(0) = 2 \)
   c. \( f(2) = 2(2) + 1 = 5 \)

27. a. \( f(1.2) = \text{int}(2(1.2)) = \text{int}(2.4) = 2 \)
   b. \( f(1.6) = \text{int}(2(1.6)) = \text{int}(3.2) = 3 \)
   c. \( f(-1.8) = \text{int}(2(-1.8)) = \text{int}(-3.6) = -4 \)

29. \( f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases} \)
   a. Domain: \( \{x \mid x \text{ is any real number}\} \)
   b. \( x\)-intercept: none
   y-intercept: \((0,1)\)
   c. Graph:

31. \( f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases} \)
   a. Domain: \( \{x \mid x \text{ is any real number}\} \)
   b. \( x\)-intercept: none
   y-intercept: \((0,3)\)
   c. Graph:

33. \( f(x) = \begin{cases} x + 3 & \text{if } -2 \leq x < 1 \\ -x + 2 & \text{if } x > 1 \end{cases} \)
   a. Domain: \( \{x \mid x \geq -2\} \)
   b. \( x\)-intercept: \((2, 0)\)
   y-intercept: \((0, 3)\)
   c. Graph:

35. \( f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \)
   a. Domain: \( \{x \mid x \text{ is any real number}\} \)
   b. \( x\)-intercepts: \((-1,0), (0,0)\)
   y-intercept: \((0,0)\)
c. Graph:

\[ y \begin{cases} \frac{x+3}{x-2} & \text{if } \frac{1}{2} < x < 2 \\ \frac{1}{x} & \text{if } 0 < x < \frac{1}{2} \\ 1 & \text{if } x = 0 \\ \frac{1}{x} & \text{if } x < 0 \end{cases} \]

d. Range: \( \{ y \mid y > 0 \} \)

37. \( f(x) = \begin{cases} \frac{1}{x} & \text{if } x = 0 \\ x^3 & \text{if } x > 0 \end{cases} \)

a. Domain: \( \{ x \mid x \leq -2 \} \)

b. \( x \)-intercept: none

y-intercept: \((0, 1)\)

c. Graph:

\[ y \begin{cases} \frac{1}{x} & \text{if } -2 \leq x < 0 \\ \frac{1}{x} & \text{if } x > 0 \end{cases} \]

d. Range: \( \{ y \mid y \text{ is any real number} \} \)

39. \( f(x) = 2 \cdot \text{int}(x) \)

a. Domain: \( \{ x \mid x \text{ is any real number} \} \)

b. \( x \)-intercepts: all ordered pairs \((x, 0)\) when \(0 \leq x < 1\).

y-intercept: \((0, 0)\)

c. Graph:

d. Range: \( \{ y \mid y > 0 \} \)

41. \( f(x) = \begin{cases} -x & \text{if } -1 \leq x < 0 \\ \frac{1}{2} x & \text{if } 0 < x \leq 2 \end{cases} \)

43. \( f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ -x + 2 & \text{if } 0 < x \leq 2 \end{cases} \)

45. \( C = \begin{cases} \frac{35}{10} & \text{if } 0 < x \leq 300 \\ 0.40x - 85 & \text{if } x > 300 \end{cases} \)

a. \( C(200) = \$35.00 \)

b. \( C(365) = 0.40(365) - 85 = \$61.00 \)

c. \( C(301) = 0.40(301) - 85 = \$35.40 \)

47. a. Charge for 50 therms:

\[
C = 9.45 + 0.36375(50) + 0.6338(50) \\
= \$59.33
\]

b. Charge for 500 therms:

\[
C = 9.45 + 0.36375(50) + 0.6338\left(\frac{x-50}{50}\right) \\
= 9.45 + 0.99755x \\
= \$396.04
\]

c. For \( 0 \leq x \leq 50 \):

\[
C = 9.45 + 0.36375x + 0.6338x \\
= 9.45 + 0.99755x
\]

For \( x > 50 \):

\[
C = 9.45 + 0.36375(x - 50) + 0.6338x \\
= 9.45 + 18.1875 + 0.11445x - 5.7225 \\
= 21.915 + 0.74825x
\]

The monthly charge function:

\[
C = \begin{cases} 9.45 + 0.99755x & \text{for } 0 \leq x \leq 50 \\ 21.915 + 0.74825x & \text{for } x > 50 \end{cases}
\]
Chapter 2: Functions and Their Graphs

49. For schedule X:

\[ f(x) = \begin{cases} 
0.10x & \text{if } x \leq 7150 \\
715 + 0.15(x - 7150) & \text{if } 7150 < x \leq 29,050 \\
4000 + 0.25(x - 29,050) & \text{if } 29,050 < x \leq 70,350 \\
14,325 + 0.28(x - 70,350) & \text{if } 70,350 < x \leq 146,750 \\
35,717 + 0.33(x - 146,750) & \text{if } 146,750 < x \leq 319,100 \\
92,592.50 + 0.35(x - 319,100) & \text{if } x > 319,100 
\end{cases} \]

51. a. Let \( x \) represent the number of miles and \( C \) be the cost of transportation.

\[ C(x) = \begin{cases} 
0.50x & \text{if } 0 \leq x \leq 100 \\
0.50(100) + 0.40(x - 100) & \text{if } 100 < x \leq 400 \\
0.50(100) + 0.40(300) + 0.25(x - 400) & \text{if } 400 < x \leq 800 \\
0.50(100) + 0.40(300) + 0.25(400) + 0(x - 800) & \text{if } 800 < x \leq 960 \\
0.50x & \text{if } 0 \leq x \leq 100 \\
10 + 0.40x & \text{if } 100 < x \leq 400 \\
70 + 0.25x & \text{if } 400 < x \leq 800 \\
270 & \text{if } 800 < x \leq 960 
\end{cases} \]

b. For hauls between 100 and 400 miles the cost is: \( C(x) = 10 + 0.40x \).

c. For hauls between 400 and 800 miles the cost is: \( C(x) = 70 + 0.25x \).
53. Let \( x \) = the amount of the bill in dollars. The minimum payment due is given by
\[
f(x) = \begin{cases} 
  x & \text{if } x < 10 \\
  10 & \text{if } 10 \leq x < 500 \\
  30 & \text{if } 500 \leq x < 1000 \\
  50 & \text{if } 1000 \leq x < 1500 \\
  70 & \text{if } 1500 \leq x 
\end{cases}
\]

55. a. \( W = 10^\circ C \)

b. \[ W = 33 - \frac{(10.45 + 10\sqrt{5} - 5)(33 - 10)}{22.04} \approx 3.98^\circ C \]

c. \[ W = 33 - \frac{(10.45 + 10\sqrt{15} - 15)(33 - 10)}{22.04} \approx -2.67^\circ C \]

d. \( W = 33 - 1.5958(33 - 10) = -3.7^\circ C \)

e. When \( 0 \leq v < 1.79 \), the wind speed is so small that there is no effect on the temperature.

f. For each drop of 1˚ in temperature, the wind chill factor drops approximately 1.6˚C.

When the wind speed exceeds 20, there is a constant drop in temperature. That is, the windchill depends only on the temperature.

57. Each graph is that of \( y = x^2 \), but shifted vertically.

If \( y = x^2 + k \), \( k > 0 \), the shift is up \( k \) units; if \( y = x^2 + k \), \( k < 0 \), the shift is down \( |k| \) units.

The graph of \( y = x^2 - 4 \) is the same as the graph of \( y = x^2 \), but shifted down 4 units. The graph of \( y = x^2 + 5 \) is the graph of \( y = x^2 \), but shifted up 5 units.

59. Each graph is that of \( y = |x| \), but either compressed or stretched vertically.

If \( y = k|x| \) and \( k > 1 \), the graph is stretched; if \( y = k|x| \) and \( 0 < k < 1 \), the graph is compressed.

The graph of \( y = \frac{1}{4} |x| \) is the same as the graph of \( y = |x| \), but compressed. The graph of \( y = 5|x| \) is the same as the graph of \( y = |x| \), but stretched.
61. The graph of \( y = \sqrt{-x} \) is the reflection about the \( y\)-axis of the graph of \( y = \sqrt{x} \).

The graph of \( y = f(-x) \) is the reflection about the \( y\)-axis of the graph of \( y = f(x) \).

63. For the graph of \( y = x^n \), \( n \) a positive even integer, as \( n \) increases, the graph of the function is narrower for \( x > 1 \) and flatter for \( x < 1 \).

65. Yes, it is a function.

\[
f(x) = \begin{cases} 
1 & \text{if } x \text{ is rational} \\
0 & \text{if } x \text{ is irrational} 
\end{cases}
\]

\( \{x \mid x \text{ is any real number} \} \)

Range = \{0, 1\}

\( y\)-intercept: \( x = 0 \Rightarrow x \) is rational \( \Rightarrow y = 1 \)

So the \( y\)-intercept is \((0, 1)\).

\( x\)-intercept: \( y = 0 \Rightarrow x \) is irrational So the graph has infinitely many \( x\)-intercepts, namely, there is an \( x\)-intercept at each irrational value of \( x \).

\( f(-x) = 1 = f(x) \) when \( x \) is rational;

\( f(-x) = 0 = f(x) \) when \( x \) is irrational, So \( f \) is even.

The graph of \( f \) consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the \( x\)-axis, and the other is located along the \( x\)-axis.

**Section 2.6**

1. horizontal; right
3. \(-5\), \(-2\), and 2
5. False; to obtain the graph of \( y = f(x + 2) - 3 \) you shift the graph of \( y = f(x) \) to the left 2 units and down 3 units.
7. B
9. H
11. I
13. L
15. F
17. G
19. \( y = (x - 4)^3 \)
21. \( y = x^3 + 4 \)
23. \( y = (-x)^3 = -x^3 \)
25. \( y = 4x^3 \)
27. (1) \( y = \sqrt{x} + 2 \)
   (2) \( y = -\sqrt{x} + 2 \)
   (3) \( y = -(-\sqrt{x} + 2) = -\sqrt{x} - 2 \)
29. (1) \( y = -\sqrt{x} \)
   (2) \( y = -\sqrt{x} + 2 \)
   (3) \( y = -\sqrt{x} + 3 + 2 \)
31. (c); To go from \( y = f(x) \) to \( y = -f(x) \) we reflect about the \( x\)-axis. This means we change the sign of the \( y\)-coordinate for each point on the graph of \( y = f(x) \). Thus, the point \((3, 0)\) would remain the same.
33. (c); To go from \( y = f(x) \) to \( y = 2f(x) \), we multiply the \( y\)-coordinate of each point on the graph of \( y = f(x) \) by 2. Thus, the point \((0, 3)\) would become \((0, 6)\).
35. \( f(x) = x^2 - 1 \)
   Using the graph of \( y = x^2 \), vertically shift downward 1 unit.

37. \( g(x) = x^3 + 1 \)
   Using the graph of \( y = x^3 \), vertically shift upward 1 unit.

39. \( h(x) = \sqrt{x - 2} \)
   Using the graph of \( y = \sqrt{x} \), horizontally shift to the right 2 units.

41. \( f(x) = (x - 1)^3 + 2 \)
   Using the graph of \( y = x^3 \), horizontally shift to the right 1 unit, then vertically shift up 2 units.

43. \( g(x) = 4\sqrt{x} \)
   Using the graph of \( y = \sqrt{x} \), vertically stretch by a factor of 4.

45. \( h(x) = \frac{1}{2x} = \left(\frac{1}{2}\right)\left(\frac{1}{x}\right) \)
   Using the graph of \( y = \frac{1}{x} \), vertically compress by a factor of \( \frac{1}{2} \).
47. \( f(x) = -\sqrt[3]{x} \)
Reflect the graph of \( y = \sqrt[3]{x} \), about the x-axis.

49. \( g(x) = |x| \)
Reflect the graph of \( y = |x| \) about the y-axis.

51. \( h(x) = -x^3 + 2 \)
Reflect the graph of \( y = x^3 \) about the x-axis, then shift vertically upward 2 units.

53. \( f(x) = 2(x + 1)^2 - 3 \)
Using the graph of \( y = x^2 \), horizontally shift to the left 1 unit, vertically stretch by a factor of 2, and vertically shift downward 3 units.

55. \( g(x) = \sqrt{x-2} + 1 \)
Using the graph of \( y = \sqrt{x} \), horizontally shift to the right 2 units and vertically shift upward 1 unit.

57. \( h(x) = \sqrt{-x} - 2 \)
Reflect the graph of \( y = \sqrt{x} \) about the y-axis and vertically shift downward 2 units.
59. \( f(x) = -(x+1)^3 - 1 \)
Using the graph of \( y = x^3 \), horizontally shift to the left 1 unit, reflect the graph about the \( x \)-axis, and vertically shift downward 1 unit.

61. \( g(x) = 2|1-x| = 2|-(1+x)| = 2|x-1| \)
Using the graph of \( y = |x| \), horizontally shift to the right 1 unit, and vertically stretch by a factor of 2.

63. \( h(x) = 2\text{int}(x-1) \)
Using the graph of \( y = \text{int}(x) \), horizontally shift to the right 1 unit, and vertically stretch by a factor of 2.

65. a. \( F(x) = f(x) + 3 \)
Shift up 3 units.

b. \( G(x) = f(x + 2) \)
Shift left 2 units.

c. \( P(x) = -f(x) \)
Reflect about the \( x \)-axis.
d. \[ H(x) = f(x + 1) - 2 \]
Shift left 1 unit and shift down 2 units.

\[ (-1, 0) \quad (1, 0) \quad (3, -2) \]

e. \[ Q(x) = \frac{1}{2} f(x) \]
Compress vertically by a factor of \( \frac{1}{2} \).

\[ (0, 1) \quad (2, 1) \quad (-4, -1) \quad (4, 0) \]

f. \[ g(x) = f(-x) \]
Reflect about the y-axis.

\[ (-2, 2) \quad (0, 2) \quad (-4, 0) \quad (4, -2) \]

g. \[ h(x) = f(2x) \]
Compress horizontally by a factor of \( \frac{1}{2} \).

\[ (0, 2) \quad (1, 2) \quad (-2, -2) \]

67. a. \[ F(x) = f(x) + 3 \]
Shift up 3 units.

\[ (-\pi, 3) \quad (-\frac{\pi}{2}, 2) \quad (\pi, 3) \]

b. \[ G(x) = f(x + 2) \]
Shift left 2 units.

\[ (-\pi - 2, 0) \quad (\pi - 2, 0) \quad (-\frac{\pi}{2}, -2) \quad (\frac{\pi}{2} - 2, -1) \]
c. \( P(x) = -f(x) \)  
Reflect about the \( x \)-axis.

\[ (-\frac{\pi}{2}, 1) \]  
\[ (\frac{\pi}{2}, -1) \]

69. a. The graph of \( y = f(x + 2) \) is the same as the graph of \( y = f(x) \), but shifted 2 units to the left. Therefore, the x-intercepts are \(-7\) and \(1\).

b. The graph of \( y = f(x - 2) \) is the same as the graph of \( y = f(x) \), but shifted 2 units to the right. Therefore, the x-intercepts are \(-3\) and \(5\).

c. The graph of \( y = 4f(x) \) is the same as the graph of \( y = f(x) \), but stretched vertically by a factor of 4. Therefore, the x-intercepts are still \(-5\) and \(3\) since the y-coordinate of each is 0.

d. The graph of \( y = f(-x) \) is the same as the graph of \( y = f(x) \), but reflected about the \( y \)-axis. Therefore, the x-intercepts are 5 and \(-3\).
71. a. The graph of \( y = f(x + 2) \) is the same as the graph of \( y = f(x) \), but shifted 2 units to the left. Therefore, the graph of \( f(x + 2) \) is increasing on the interval \((-3,3)\).

b. The graph of \( y = f(x - 5) \) is the same as the graph of \( y = f(x) \), but shifted 5 units to the right. Therefore, the graph of \( f(x - 5) \) is increasing on the interval \((4,10)\).

c. The graph of \( y = -f(x) \) is the same as the graph of \( y = f(x) \), but reflected about the x-axis. Therefore, we can say that the graph of \( y = -f(x) \) must be decreasing on the interval \((-1,5)\).

d. The graph of \( y = f(-x) \) is the same as the graph of \( y = f(x) \), but reflected about the y-axis. Therefore, we can say that the graph of \( y = f(-x) \) must be decreasing on the interval \((-5,1)\).

73. a. \( y = |f(x)| \)

b. \( y = f(|x|) \)

75. \( f(x) = x^2 + 2x \\
    f(x) = (x^2 + 2x + 1) - 1 \\
    f(x) = (x + 1)^2 - 1 \\
Using \( f(x) = x^2 \), shift left 1 unit and shift down 1 unit.

77. \( f(x) = x^2 - 8x + 1 \\
    f(x) = (x^2 - 8x + 16) + 1 - 16 \\
    f(x) = (x - 4)^2 - 15 \\
Using \( f(x) = x^2 \), shift right 4 units and shift down 15 units.
79. \( f(x) = x^2 + x + 1 \)
\[
\begin{align*}
  f(x) &= \left( x^2 + x + \frac{1}{4} \right) + 1 - \frac{1}{4} \\
  f(x) &= \left( x + \frac{1}{2} \right)^2 + \frac{3}{4}
\end{align*}
\]
Using \( f(x) = x^2 \), shift left \( \frac{1}{2} \) unit and shift up \( \frac{3}{4} \) unit.

81. \( f(x) = 2x^2 - 12x + 19 \)
\[
\begin{align*}
  f(x) &= 2(x^2 - 6x) + 19 \\
  &= 2(x^2 - 6x + 9) + 19 - 18 \\
  &= 2(x - 3)^2 + 1
\end{align*}
\]
Using \( f(x) = x^2 \), shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.

83. \( f(x) = -3x^2 - 12x - 17 \)
\[
\begin{align*}
  &= -3\left( x^2 + 4x \right) - 17 \\
  &= -3\left( x^2 + 4x + 4 \right) - 17 + 12 \\
  &= -3(x + 2)^2 - 5
\end{align*}
\]
Using \( f(x) = x^2 \), shift left 2 units, stretch vertically by a factor of 3, reflect about the x-axis, and shift down 5 units.

85. \( y = (x - c)^2 \)
If \( c = 0 \), \( y = x^2 \).
If \( c = 3 \), \( y = (x - 3)^2 \); shift right 3 units.
If \( c = -2 \), \( y = (x + 2)^2 \); shift left 2 units.

87. \( F = \frac{9}{5} C + 32 \)
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89. a. \( p(x) = -0.05x^2 + 100x - 2000 \)

b. Select the 10% tax since the profits are higher.

c. The graph of \( Y_1 \) is obtained by shifting the graph of \( p(x) \) vertically down 10,000 units. The graph of \( Y_2 \) is obtained by multiplying the \( y \)-coordinate of the graph of \( p(x) \) by 0.9. Thus, \( Y_2 \) is the graph of \( p(x) \) vertically compressed by a factor of 0.9.

d. Select the 10% tax since the graph of \( Y_1 = 0.9 \cdot p(x) = -0.05x^2 + 100x - 6800 \) for all \( x \) in the domain.

91. a. \( y_1 = x + 1; \quad y_2 = |x + 1| \)

b. \( y_1 = 4 - x^2; \quad y_2 = |4 - x^2| \)

c. \( y_1 = x^3 + x; \quad y_2 = |x^3 + x| \)

d. Any part of the graph of \( y = f(x) \) that lies below the \( x \)-axis is reflected about the \( x \)-axis to obtain the graph of \( y = |f(x)| \).
Section 2.7

1. \( V = \pi r^2 h, \quad h = 2r \Rightarrow V(r) = \pi r^2 \cdot (2r) = 2\pi r^3 \)

3. a. \( R(x) = x \left( -\frac{1}{6} x + 100 \right) = -\frac{1}{6} x^2 + 100x \)

b. \( R(200) = -\frac{1}{6} (200)^2 + 100(200) \)
   \[ = -\frac{20,000}{3} + 20,000 \]
   \[ = \frac{40,000}{3} \approx 13,333.33 \]

c.

\[ \begin{array}{c}
\text{600} \\
\text{16000} \\
\text{0} \end{array} \]

\[ \begin{array}{c}
\text{0} \\
\text{600} \end{array} \]

d. \( x = 300 \) maximizes revenue
   \[ R(300) = -\frac{1}{6} (300)^2 + 100(300) \]
   \[ = -15,000 + 30,000 \]
   \[ = 15,000 \]
   The maximum revenue is $15,000.

e. \( p = -\frac{1}{6} (300) + 100 = -50 + 100 = 50 \)
   maximizes revenue

5. a. If \( x = -5p + 100 \), then \( p = \frac{100 - x}{5} \).
   \[ R(x) = x \left( \frac{100 - x}{5} \right) = -\frac{1}{5} x^2 + 20x \]

b. \( R(15) = -\frac{1}{5} (15)^2 + 20(15) \)
   \[ = -45 + 300 = 255 \]

d. \( x = 50 \) maximizes revenue
   \[ R(50) = -\frac{1}{5} (50)^2 + 20(50) \]
   \[ = -500 + 1000 = 500 \]
   The maximum revenue is $500.

e. \( p = \frac{100 - 50}{5} = \frac{50}{5} = 10 \)
   maximizes revenue.

7. a. Let \( x = \) width and \( y = \) length of the rectangular area.
   \[ P = 2x + 2y = 400 \]
   \[ y = \frac{400 - 2x}{2} = 200 - x \]
   Then
   \[ A(x) = (200 - x)x \]
   \[ = 200x - x^2 \]
   \[ = -x^2 + 200x \]

b. We need
   \( x > 0 \) and \( y > 0 \Rightarrow 200 - x > 0 \Rightarrow 200 > x \)
   So the domain of \( A \) is \( \{x | 0 < x < 200 \} \)

c. \( x = 100 \) yards maximizes area

\[ \begin{array}{c}
10000 \\
0 \\
200 \end{array} \]

\[ \begin{array}{c}
0 \\
1000 \end{array} \]
9. a. The distance \( d \) from \( P \) to the origin is 
\[ d = \sqrt{x^2 + y^2}. \]
Since \( P \) is a point on the graph of \( y = x^2 - 8 \), we have:
\[ d(x) = \sqrt{x^2 + (x^2 - 8)^2} = \sqrt{x^4 - 15x^2 + 64} \]
b. \( d(0) = \sqrt{0^4 - 15(0)^2 + 64} = \sqrt{64} = 8 \)
c. \( d(1) = \sqrt{(1)^4 - 15(1)^2 + 64} = \sqrt{1 - 15 + 64} = \sqrt{50} = 5\sqrt{2} \approx 7.07 \)
d. \( d \) is smallest when \( x \approx -2.74 \) and when \( x \approx 2.74 \).

e. \( d \) is smallest when \( x \approx -2.74 \) and when \( x \approx 2.74 \).

11. a. The distance \( d \) from \( P \) to the point \((1, 0)\) is 
\[ d = \sqrt{(x-1)^2 + y^2}. \]
Since \( P \) is a point on the graph of \( y = \sqrt{x} \), we have:
\[ d(x) = \sqrt{(x-1)^2 + (\sqrt{x})^2} = \sqrt{x^2 - x + 1} \]
where \( x \geq 0 \).

13. By definition, a triangle has area 
\[ A = \frac{1}{2} bh, \quad b = \text{base}, \quad h = \text{height}. \]
From the figure, we know that \( b = x \) and \( h = y \). Expressing the area of the triangle as a function of \( x \), we have:
\[ A(x) = \frac{1}{2} xy = \frac{1}{2} x \left( x^2 \right) = \frac{1}{2} x^4. \]

15. a. \( A(x) = xy = x \left( 16 - x^2 \right) = -x^3 + 16x \)
b. Domain: \( \{ x \mid 0 < x < 4 \} \)
c. The area is largest when \( x \) is approximately 2.31.

17. a. In Quadrant I, \( x^2 + y^2 = 4 \rightarrow y = \sqrt{4 - x^2} \)
\[ A(x) = (2x)(2y) = 4x\sqrt{4-x^2} \]
b. \( p(x) = 2(2x) + 2(2y) = 4x + 4\sqrt{4-x^2} \)
c. Graphing the area equation:
The area is largest when $x$ is roughly 1.41.

d. Graphing the perimeter equation:

The perimeter is largest when $x$ is approximately 1.41.

19. a. $C =$ circumference, $TA =$ total area, $r =$ radius, $x =$ side of square

$$C = 2\pi r = 10 - 4x \implies r = \frac{5-2x}{\pi}$$

Total Area = area square + area circle

$$TA(x) = x^2 + \pi r^2$$

$$= x^2 + \pi \left(\frac{5-2x}{\pi}\right)^2$$

$$= x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

$10 - 4x > 0$ and $x > 0$

$-4x > -10$ and $x > 0$

$x < 2.5$ and $x > 0$

Domain: $\{x | 0 < x < 2.5\}$

c. The total area is smallest when $x$ is approximately 1.40 meters.

21. a. Since the wire of length $x$ is bent into a circle, the circumference is $x$. Therefore, $C(x) = x$.

b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi \left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$
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27. \( d^2 = d_1^2 + d_2^2 \)
\[ d^2 = (2 - 3t)^2 + (3 - 4t)^2 \]
\[ d(t) = \sqrt{(2 - 3t)^2 + (3 - 4t)^2} \]
\[ = \sqrt{4 - 12t + 9t^2 + 9 - 24t + 16t^2} \]
\[ = \sqrt{25t^2 - 36t + 13} \]
\[ d_1 = 2 - 3t \]

b. The distance is smallest at \( t \approx 0.072 \) hours.

29. \( r = \) radius of cylinder, \( h = \) height of cylinder, \( V = \) volume of cylinder

By similar triangles:
\[ \frac{H}{r} = \frac{H - h}{r} \]
\[ Hr = Rh - Rh \]
\[ R = H - Hr \]
\[ h = \frac{RH}{r} = \frac{H - Hr}{r} \]
\[ V = \pi r^2 h = \pi r^2 \left( H - \frac{Hr}{r} \right) = Hr^2 \left( 1 - \frac{r}{R} \right) \]

31. a. The time on the boat is given by \( \frac{d_1}{3} \). The time on land is given by \( \frac{12-x}{5} \).

b. Domain: \( \{ x \mid 0 \leq x \leq 12 \} \)

c. \( T(4) = \frac{12 - 4 + \sqrt{4^2 + 4}}{5} = \frac{8 + \sqrt{20}}{5} = \frac{8 + 2\sqrt{5}}{5} \approx 3.09 \text{ hours} \)

d. \( T(8) = \frac{12 - 8 + \sqrt{8^2 + 4}}{5} = \frac{4 + \sqrt{68}}{5} = \frac{4 + 9.38}{5} \approx 3.55 \text{ hours} \)

Chapter 2 Review

1. This relation represents a function.
   Domain = \{ -1, 2, 4 \}; Range = \{ 0, 3 \}.

3. \( f(x) = \frac{3x}{x^2 - 1} \)
   a. \( f(2) = \frac{3(2)}{(2)^2 - 1} = \frac{6}{4 - 1} = \frac{6}{3} = 2 \)
   b. \( f(-2) = \frac{3(-2)}{(-2)^2 - 1} = \frac{-6}{4 - 1} = \frac{-6}{3} = -2 \)
   c. \( f(-x) = \frac{3(-x)}{(-x)^2 - 1} = \frac{-3x}{x^2 - 1} \)
   d. \( -f(x) = -\left( \frac{3x}{x^2 - 1} \right) = \frac{-3x}{x^2 - 1} \)
   e. \( f(x - 2) = \frac{3(x - 2)}{(x - 2)^2 - 1} \)
      \[ = \frac{3x - 6}{x^2 - 4x + 4 - 1} \]
      \[ = \frac{3x - 6}{x^2 - 4x + 3} \]
   f. \( f(2x) = \frac{3(2x)}{(2x)^2 - 1} = \frac{6x}{4x^2 - 1} \)
5. \( f(x) = \sqrt{x^2 - 4} \)

a. \( f(2) = \sqrt{2^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0 \)

b. \( f(-2) = \sqrt{(-2)^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0 \)

c. \( f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4} \)

d. \( -f(x) = -\sqrt{x^2 - 4} \)

e. \( f(x-2) = \sqrt{(x-2)^2 - 4} \)
\[= \sqrt{x^2 - 4x + 4 - 4} \]
\[= \sqrt{x^2 - 4x} \]

f. \( f(2x) = \sqrt{(2x)^2 - 4} = \sqrt{4x^2 - 4} \)
\[= \sqrt{4(x^2 - 1)} = 2\sqrt{x^2 - 1} \]

7. \( f(x) = \frac{x^2 - 4}{x^2} \)

a. \( f(2) = \frac{2^2 - 4}{2^2} = \frac{4 - 4}{4} = \frac{0}{4} = 0 \)

b. \( f(-2) = \frac{(-2)^2 - 4}{(-2)^2} = \frac{4 - 4}{4} = \frac{0}{4} = 0 \)

c. \( f(-x) = \frac{(-x)^2 - 4}{(-x)^2} = \frac{x^2 - 4}{x^2} \)

d. \( -f(x) = - \left( \frac{x^2 - 4}{x^2} \right) = \frac{4 - x^2}{x^2} = \frac{-x^2 - 4}{x^2} \)

e. \( f(x-2) = \frac{(x-2)^2 - 4}{(x-2)^2} = \frac{x^2 - 4x + 4 - 4}{(x-2)^2} \)
\[= \frac{x^2 - 4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2} \]

f. \( f(2x) = \frac{(2x)^2 - 4}{(2x)^2} = \frac{4x^2 - 4}{4x^2} \)
\[= \frac{4(x^2 - 1)}{4x^2} = \frac{x^2 - 1}{x^2} \]

9. \( f(x) = \frac{x}{x^2 - 9} \)

The denominator cannot be zero:
\[x^2 - 9 \neq 0\]
\[(x + 3)(x - 3) \neq 0\]
\[x \neq -3 \text{ or } 3\]
Domain: \( \{ x | x \neq -3, x \neq 3 \} \)

11. \( f(x) = \sqrt{2 - x} \)

The radicand must be non-negative:
\[2 - x \geq 0\]
\[x \leq 2\]
Domain: \( \{ x | x \leq 2 \} \text{ or } (-\infty, 2] \)

13. \( f(x) = \sqrt{\frac{x}{x}} \)

The radicand must be non-negative and the denominator cannot be zero: \( x > 0 \)
Domain: \( \{ x | x > 0 \} \text{ or } (0, \infty) \)

15. \( f(x) = \frac{x}{x^2 + 2x - 3} \)

The denominator cannot be zero:
\[x^2 + 2x - 3 \neq 0\]
\[(x + 3)(x - 1) \neq 0\]
\[x \neq -3 \text{ or } 1\]
Domain: \( \{ x | x \neq -3, x \neq 1 \} \)

17. \( f(x) = 2 - x \quad g(x) = 3x + 1 \)

\((f + g)(x) = f(x) + g(x) = 2 - x + 3x + 1 = 2x + 3\)
Domain: \( \{ x | x \text{ is any real number} \} \)

\((f - g)(x) = f(x) - g(x) = 2 - x - (3x + 1) = 2 - x - 3x - 1 = -4x + 1\)
Domain: \( \{ x | x \text{ is any real number} \} \)
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\[(f \cdot g)(x) = f(x) \cdot g(x) = (2 - x)(3x + 1) = 6x + 2 - 3x^2 - x = -3x^2 + 5x + 2\]

Domain: \(\{x \mid x \text{ is any real number}\}\)

\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2 - x}{3x + 1}
\]

\[3x + 1 \neq 0 \implies x \neq -\frac{1}{3}\]

Domain: \(\left\{x \mid x \neq -\frac{1}{3}\right\}\)

19. \(f(x) = 3x^2 + x + 1\)

\[
g(x) = 3x
\]

\[(f + g)(x) = f(x) + g(x) = 3x^2 + x + 1 + 3x = 3x^2 + 4x + 1\]

Domain: \(\{x \mid x \text{ is any real number}\}\)

\[(f - g)(x) = f(x) - g(x) = 3x^2 + x + 1 - 3x = 3x^2 - 2x + 1\]

Domain: \(\{x \mid x \text{ is any real number}\}\)

\[(f \cdot g)(x) = f(x) \cdot g(x) = (3x^2 + x + 1)(3x) = 9x^3 + 3x^2 + 3x\]

Domain: \(\{x \mid x \text{ is any real number}\}\)

\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2 + x + 1}{3x}
\]

\[3x \neq 0 \implies x \neq 0\]

Domain: \(\{x \mid x \neq 0\}\)

21. \(f(x) = \frac{x + 1}{x - 1}\)

\[
g(x) = \frac{1}{x}
\]

\[
(f + g)(x) = f(x) + g(x) = \frac{x + 1}{x - 1} + \frac{1}{x} = \frac{x(x + 1) + 1(x - 1)}{x(x - 1)} = \frac{x^2 + x + x - 1}{x(x - 1)} = \frac{x^2 + 2x - 1}{x(x - 1)}
\]

Domain: \(\{x \mid x \neq 0, x \neq 1\}\)

\[
(f - g)(x) = f(x) - g(x) = \frac{x + 1}{x - 1} - \frac{1}{x} = \frac{x(x + 1) - 1(x - 1)}{x(x - 1)} = \frac{x^2 + x - x + 1}{x(x - 1)} = \frac{x^2 + 1}{x(x - 1)}
\]

Domain: \(\{x \mid x \neq 0, x \neq 1\}\)

\[
(f \cdot g)(x) = f(x) \cdot g(x) = \left(\frac{x + 1}{x - 1}\right) \left(\frac{1}{x}\right) = \frac{x + 1}{x(x - 1)}
\]

Domain: \(\{x \mid x \neq 0, x \neq 1\}\)

\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + 1}{x - 1} \cdot \frac{1}{x} = \frac{x + 1}{x} \cdot \frac{x}{x - 1} = \frac{x(x + 1)}{x - 1}
\]

Domain: \(\{x \mid x \neq 0, x \neq 1\}\)
23. \[ f(x) = -2x^2 + x + 1 \]

\[
\frac{f(x+h) - f(x)}{h} = \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2 + x + 1)}{h}
\]

\[
= \frac{-2(x^2 + 2xh + h^2) + x + h + 1 + 2x^2 - x - 1}{h}
\]

\[
= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}
\]

\[
= \frac{4x - 2h^2 + h}{h} = \frac{h(-4x - 2h + 1)}{h}
\]

\[
= -4x - 2h + 1
\]

25. a. Domain: \( \{ x \mid -4 \leq x \leq 3 \} \)

Range: \( \{ y \mid -3 \leq y \leq 3 \} \)

b. x-intercept: \((0,0)\); y-intercept: \((0,0)\)

c. \( f(-2) = -1 \)

d. \( f(x) = -3 \) when \( x = -4 \)

e. \( f(x) > 0 \) when \( 0 < x \leq 3 \)

f. To graph \( y = f(x-3) \), shift the graph of \( f \) horizontally 3 units to the right.

g. To graph \( y = f\left(\frac{1}{2}x\right) \), stretch the graph of \( f \) horizontally by a factor of 2.

h. To graph \( y = -f(x) \), reflect the graph of \( f \) vertically about the \( y \)-axis.

27. a. Domain: \( \{ x \mid -4 \leq x \leq 4 \} \)

Range: \( \{ y \mid -3 \leq y \leq 1 \} \)

b. Increasing: \((-4,-1)\) and \((3,4)\);
Decreasing: \((-1,3)\)

c. Local minimum is \(-3\) when \( x = 3 \);
Local maximum is \(1\) when \( x = -1 \).
Note that \( x = 4 \) and \( x = -4 \) do not yield local extrema because there is no open interval that contains either value.

d. The graph is not symmetric with respect to the \( x \)-axis, the \( y \)-axis or the origin.

e. The function is neither even nor odd.

f. x-intercepts: \((-2,0)\), \((0,0)\), \((4,0)\), y-intercept: \((0,0)\)
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29. \( f(x) = x^3 - 4x \)
\( f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x \)
\( = -f(x) \)

\( f \) is odd.

31. \( h(x) = \frac{1}{x^2} + \frac{1}{x^2} + 1 \)
\( h(-x) = \frac{1}{(-x)^2} + \frac{1}{(-x)^2} + 1 = h(x) \)

\( h \) is even.

33. \( G(x) = 1 - x + x^3 \)
\( G(-x) = 1 - (-x) + (-x)^3 \)
\( = 1 + x - x^3 \neq -G(x) \) or \( G(x) \)

\( G \) is neither even nor odd.

35. \( f(x) = \frac{x}{1+x^2} \)
\( f(-x) = \frac{-x}{1+(-x)^2} = -f(x) \)

\( f \) is odd.

37. \( f(x) = 2x^3 - 5x + 1 \) on the interval \((-3,3)\)
Use MAXIMUM and MINIMUM on the graph of \( y_1 = 2x^3 - 5x + 1 \).

39. \( f(x) = 2x^4 - 5x^3 + 2x + 1 \) on the interval \((-2,3)\)
Use MAXIMUM and MINIMUM on the graph of \( y_1 = 2x^4 - 5x^3 + 2x + 1 \).

41. \( f(x) = 8x^2 - x \)

\[ \frac{f(2) - f(1)}{2 - 1} = \frac{8(2)^2 - 2 - (8(1)^2 - 1)}{1} \]
\[ = 32 - 2 - (7) = 23 \]

\[ \frac{f(1) - f(0)}{1 - 0} = \frac{8(1)^2 - 1 - (8(0)^2 - 0)}{1} \]
\[ = 8 - 1 - (0) = 7 \]

\[ \frac{f(4) - f(2)}{4 - 2} = \frac{8(4)^2 - 4 - (8(2)^2 - 2)}{2} \]
\[ = \frac{128 - 4 - (30)}{2} = \frac{94}{2} = 47 \]

43. \( f(x) = 2 - 5x \)
\( \frac{f(x) - f(2)}{x - 2} = \frac{2 - 5x - (-8)}{x - 2} = \frac{-5x + 10}{x - 2} \)
\[ = \frac{-5(x - 2)}{x - 2} = -5 \]
45. \( f(x) = 3x - 4x^2 \)

\[
\frac{f(x) - f(2)}{x - 2} = \frac{3x - 4x^2 - (-10)}{x - 2} = \frac{-4x^2 + 3x + 10}{x - 2} = -\frac{(4x^2 - 3x - 10)}{x - 2} = -\frac{(4x + 5)(x - 2)}{x - 2} = -4x - 5
\]

47. (b) passes the Vertical Line Test and is therefore a function.

49. \( f(x) = 2x - 5 \)

51. \( h(x) = \frac{4}{2} x - 6 \)

53. \( f(x) = |x| \)

55. \( F(x) = |x| - 4 \)

Using the graph of \( y = |x| \), vertically shift the graph downward 4 units.

57. \( g(x) = -2|x| \)

Reflect the graph of \( y = |x| \) about the x-axis and vertically stretch the graph by a factor of 2.
59. \( h(x) = \sqrt{x-1} \)

Using the graph of \( y = \sqrt{x} \), horizontally shift the graph to the right 1 unit.

Intercept: (1, 0)
Domain: \( \{x \mid x \geq 1\} \)
Range: \( \{y \mid y \geq 0\} \)

61. \( f(x) = \sqrt{1-x} = \sqrt{-1(x-1)} \)

Reflect the graph of \( y = \sqrt{x} \) about the y-axis and horizontally shift the graph to the right 1 unit.

Intercepts: (1, 0), (0,1)
Domain: \( \{x \mid x \leq 1\} \)
Range: \( \{y \mid y \geq 0\} \)

63. \( h(x) = (x-1)^2 + 2 \)

Using the graph of \( y = x^2 \), horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.

Intercepts: (0, 3)
Domain: \( \{x \mid x \text{ is any real number}\} \)
Range: \( \{y \mid y \geq 2\} \)

65. \( g(x) = 3(x-1)^3 + 1 \)

Using the graph of \( y = x^3 \), horizontally shift the graph to the right 1 unit vertically stretch the graph by a factor of 3, and vertically shift the graph up 1 unit.

Intercepts: (0,−2), \( \left(1-\frac{\sqrt[3]{9}}{3}, 0\right) \)
Domain: \( \{x \mid x \text{ is any real number}\} \)
Range: \( \{y \mid y \text{ is any real number}\} \)

67. \( f(x) = \begin{cases} 3x & \text{if } -2 < x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases} \)

a. Domain: \( \{x \mid x > -2\} \)
b. \( x\)-intercept: (0,0)
\( y\)-intercept: (0,0)
c. Graph:

d. Range: \( \{y > -6\} \)
69. \( f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \\ x & \text{if } -4 \leq x < 0 \end{cases} \)

a. Domain: \( \{ x \mid x \geq -4 \} \)

b. \( x \)-intercept: none
   \( y \)-intercept: \((0, 1)\)

c. Graph:

![Graph](image)

d. Range: \( \{ y \mid y \geq -4, y \neq 0 \} \)

71. \( f(4) = -5 \) gives the ordered pair \((4, -5)\)
    \( f(0) = 3 \) gives \( f(0) = 3 \) gives \((0, 3)\)

Finding the slope: \( m = \frac{3 - (-5)}{0 - (-4)} = \frac{8}{4} = 2 \)

Using slope-intercept form: \( f(x) = -2x + 3 \)

73. \( f(x) = \frac{Ax + 5}{6x - 2} \) and \( f(1) = 4 \)
    \( A(1) + 5 \)
    \( 6(1) - 2 \)
    \( = 4 \)
    \( = 4 \)
    \( A + 5 = 16 \)
    \( A = 11 \)

75. We have the points \((h_1, T_1) = (0, 30)\) and \((h_2, T_2) = (10000, 5)\).
    \( \text{slope} = \frac{\Delta T}{\Delta h} = \frac{5 - 30}{10000 - 0} = -25 \)
    \( 10000 \)
    \( = -0.0025 \)

Using the point-slope formula yields
    \( T - T_1 = m(h - h_1) \Rightarrow T - 30 = -0.0025(h - 0) \)
    \( T - 30 = -0.0025h \Rightarrow T = -0.0025h + 30 \)
    \( T(h) = -0.0025h + 30, \ 0 \leq h \leq 10,000 \)

77. \( S = 4\pi r^2; V = \frac{4}{3}\pi r^3 \)
    Let \( R = 2r, S_2 = \text{new surface area}, \) and \( V_2 = \text{new volume} \).
    \( S_2 = 4\pi R^2 = 4\pi (2r)^2 \)
    \( = 4\pi (4r^2) = 4(4\pi r^2) \)
    \( = 4S \)
    \( V_2 = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (2r)^3 \)
    \( = \frac{4}{3}\pi (8r^3) = 8 \left( \frac{4}{3}\pi r^3 \right) \)
    \( = 8V \)

Thus, if the radius of the sphere doubles, the surface area is 4 times as large and the volume is 8 times as large as for the original sphere.

79. \( S = kxd^3, \ x = \text{width}; \ d = \text{depth} \)
    In the diagram, \( d = \text{length of the rectangle} \).
    Therefore, we have
    \( \left( \frac{d}{2} \right)^2 + \left( \frac{x}{2} \right)^2 = 3^2 \)
    \( \frac{d^2}{4} + \frac{x^2}{4} = 9 \)
    \( d^2 + x^2 = 36 \)
    \( d = \sqrt{36 - x^2} \)
    \( S(x) = kx \left( \sqrt{36 - x^2} \right)^3 = kx \left( 36 - x^2 \right)^{3/2} \)
    Domain: \( \{ x \mid 0 < x < 6 \} \)
81. a. The relation is a function. Each HS GPA value is paired with exactly one College GPA value.

b. Scatter diagram:

![Scatter Diagram]

c. Using the LINear REGression program, the line of best fit is: \( G = 0.964x + 0.072 \)

d. As the high school GPA increases by 0.1 point, the college GPA increases by 0.0964 point.

e. \( G(x) = 0.964x + 0.072 \)

f. Domain: \( \{x \mid 0 \leq x \leq 4\} \)

g. \( G(3.23) = (0.964)(3.23) + 0.072 \approx 3.19 \)

The college GPA is approximately 3.19.

83. Let \( R \) = the revenue in dollars, and \( g \) = the number of gallons of gasoline sold.

Consider the ordered pair \((g, R)\). We can use the points \((0, 0)\) and \((13.5, 28.89)\). Now compute the slope:

\[
\text{slope} = \frac{\Delta y}{\Delta x} = \frac{28.89 - 0}{13.5 - 0} = \frac{28.89}{13.5} \approx 2.14
\]

Therefore we have the linear function \( R(g) = 2.14g + 0 = 2.14g \).

If \( g = 11.2 \), then \( R(2.14)(11.2) = 23.97 \).

85. Let \( x \) represent the length and \( y \) represent the width of the rectangle.

\[
2x + 2y = 20 \implies y = 10 - x
\]

\[
x \cdot y = 16 \implies x(10 - x) = 16
\]

Solving the area equation:

\[
10x - x^2 = 16 \implies x^2 - 10x + 16 = 0
\]

\[
(x - 8)(x - 2) = 0 \implies x = 8 \text{ or } x = 2
\]

The length and width of the rectangle are 8 feet by 2 feet.

87. \( C(x) = 4.9x^2 - 617.40x + 19,600 \);
\( a = 4.9 \), \( b = -617.40 \), \( c = 19,600 \).

Since \( a = 4.9 > 0 \), the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at \( x = 63 \).

![Minimum Marginal Cost Graph]

b. The minimum marginal cost is

\[
C\left(-\frac{b}{2a}\right) = C(63) = 4.9(63)^2 - (617.40)(63) + 19600 = 151.90
\]

89. Let \( P = (4,1) \) and \( Q = (x, y) = (x, x+1) \).

\[
d(P, Q) = \sqrt{(x - 4)^2 + (x + 1 - 1)^2}
\]

\[
\rightarrow d^2(x) = (x - 4)^2 + x^2 = x^2 - 8x + 16 + x^2 = 2x^2 - 8x + 16
\]

Since \( d^2(x) = 2x^2 - 8x + 16 \) is a quadratic function with \( a = 2 > 0 \), the vertex corresponds to the minimum value for the function.

![Minimum Distance Graph]

The vertex occurs at \( x = 2 \). Therefore the point \( Q \) on the line \( y = x + 1 \) will be closest to the point \( P = (4,1) \) when \( Q = (2,3) \).
91. a. \( x^2 h = 10 \implies h = \frac{10}{x^2} \)
\[ A(x) = 2x^2 + 4xh \]
\[ = 2x^2 + 4x \left( \frac{10}{x^2} \right) \]
\[ = 2x^2 + \frac{40}{x} \]

b. \( A(1) = 2 \cdot 1^2 + \frac{40}{1} = 2 + 40 = 42 \text{ ft}^2 \)

c. \( A(2) = 2 \cdot 2^2 + \frac{40}{2} = 8 + 20 = 28 \text{ ft}^2 \)

d. Graphing: The area is smallest when \( x \approx 2.15 \text{ feet} \).

Chapter 2 Test

1. a. \{ (2,5), (4,6), (6,7), (8,8) \}  
   This relation is a function because there are no ordered pairs that have the same first element and different second elements. 
   Domain: \{ 2, 4, 6, 8 \}  
   Range: \{ 5, 6, 7, 8 \}  

b. \{ (1,3), (4,-2), (-3,5), (1,7) \}  
   This relation is not a function because there are two ordered pairs that have the same first element but different second elements. 

c. This relation is not a function because the graph fails the vertical line test. 

d. This relation is a function because it passes the vertical line test.

2. \( f(x) = \sqrt{4 - 5x} \)
   The function tells us to take the square root of \( 4 - 5x \). Only nonnegative numbers have real square roots so we need \( 4 - 5x \geq 0 \).
   \[ 4 - 5x \geq 0 \] 
   \[ -5x \geq -4 \] 
   \[ x \leq \frac{4}{5} \]
   Domain: \( \{ x \mid x \leq \frac{4}{5} \} \)
\[ f(-1) = \sqrt{4 - 5(-1)} = \sqrt{4 + 5} = \sqrt{9} = 3 \]

3. \( g(x) = \frac{x+2}{|x+2|} \)
   The function tells us to divide \( x + 2 \) by \( |x+2| \).  
   Division by 0 is undefined, so the denominator can never equal 0. This means that \( x \neq -2 \).
   Domain: \( \{ x \mid x \neq -2 \} \)
\[ g(-1) = \frac{(-1)+2}{|(-1)+2|} = \frac{1}{1} = 1 \]

4. \( h(x) = \frac{x-4}{x^2+5x-36} \)
   The function tells us to divide \( x - 4 \) by \( x^2 + 5x - 36 \). Since division by 0 is not defined, we need to exclude any values which make the denominator 0.
   \[ x^2 + 5x - 36 = 0 \] 
   \[ (x+9)(x-4) = 0 \] 
   \[ x = -9 \text{ or } x = 4 \]
   Domain: \( \{ x \mid x \neq -9, x \neq 4 \} \)  
   (note: there is a common factor of \( x - 4 \) but we must determine the domain prior to simplifying)
\[ h(-1) = \frac{(-1)-4}{(-1)^2+5(-1)-36} = \frac{-5}{-40} = \frac{1}{8} \]
5. a. To find the domain, note that all the points on the graph will have an x-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a y-coordinate between -3 and 3, inclusive.

Domain: \( \{ x \mid -5 \leq x \leq 5 \} \)

Range: \( \{ y \mid -3 \leq y \leq 3 \} \)

b. The intercepts are (0, 2) , (-2, 0) , and (2, 0).

x-intercepts: 2, -2

y-intercept: 2

c. \( f(1) \) is the value of the function when \( x = 1 \). According to the graph, \( f(1) = 3 \).

d. Since (-5, -3) and (3, -3) are the only points on the graph for which \( y = f(x) = -3 \) , we have \( f(x) = -3 \) when \( x = -5 \) and \( x = 3 \).

e. To solve \( f(x) < 0 \) , we want to find x-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is \( \{ x \mid -5 \leq x < -2 \text{ or } 2 < x \leq 5 \} \). In interval notation we would write the solution set as \([-5, -2) \cup (2, 5]\).

6. \( f(x) = -x^4 + 2x^3 + 4x^2 - 2 \)

We set \( \text{Xmin} = -5 \) and \( \text{Xmax} = 5 \). The standard \( \text{Ymin} \) and \( \text{Ymax} \) will not be good enough to see the whole picture so some adjustment must be made.

![Graph of the function](image)

We see that the graph has a local maximum of -0.86 (rounded to two places) when \( x = -0.85 \) and another local maximum of 15.55 when \( x = 2.35 \). There is a local minimum of -2 when \( x = 0 \). Thus, we have

Local maxima: \( f(-0.85) \approx -0.86 \)

\( f(2.35) \approx 15.55 \)

Local minima: \( f(0) = -2 \)

The function is increasing on the intervals (-5, -0.85) and (0, 2.35) and decreasing on the intervals (-0.85, 0) and (2.35, 5).

7. a. \( f(x) = \begin{cases} 2x + 1 & x < -1 \\ x - 4 & x \geq -1 \end{cases} \)

To graph the function, we graph each “piece”. First we graph the line \( y = 2x + 1 \) but only keep the part for which \( x < -1 \). Then we plot the line \( y = x - 4 \) but only keep the part for which \( x \geq -1 \).

b. To find the intercepts, notice that the only piece that hits either axis is \( y = x - 4 \).

\( y = x - 4 \)

The intercepts are (0, -4) and (4, 0).

c. To find \( g(-5) \) we first note that \( x = -5 \) so we must use the first “piece” because \(-5 < -1\).

\( g(-5) = 2(-5) + 1 = -10 + 1 = -9 \)
d. To find \( g(2) \) we first note that \( x = 2 \) so we must use the second “piece” because \( 2 \geq -1 \).
\[
g(2) = 2 - 4 = -2
\]

8. The average rate of change from 3 to \( x \) is given by
\[
\frac{\Delta y}{\Delta x} = \frac{f(x) - f(3)}{x-3} \quad x \neq 3
\]
\[
= \frac{(3x^2 - 2x + 4) - (3(3)^2 - 2(3) + 4)}{x-3}
\]
\[
= \frac{3x^2 - 2x + 4 - 25}{x-3}
\]
\[
= \frac{3x^2 - 2x - 21}{x-3}
\]
\[
= \frac{(x-3)(3x+7)}{x-3}
\]
\[
= 3x + 7 \quad x \neq 3
\]

9. a. \( f - g = (2x^2 + 1) - (3x - 2) \)
\[
= 2x^2 + 1 - 3x + 2
\]
\[
= 2x^2 - 3x + 3
\]

b. \( f \cdot g = (2x^2 + 1)(3x - 2) \)
\[
= 6x^3 - 4x^2 + 3x - 2
\]

c. \( f(x+h) - f(x) \)
\[
= (2(x+h)^2 + 1) - (2x^2 + 1)
\]
\[
= (2(x^2 + 2xh + h^2) + 1) - (2x^2 + 1)
\]
\[
= 2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1
\]
\[
= 4xh + 2h^2
\]

10. a. The basic function is \( y = x^3 \) so we start with the graph of this function.

Next we shift this graph 1 unit to the left to obtain the graph of \( y = (x+1)^3 \).

Next we reflect this graph about the x-axis to obtain the graph of \( y = -(x+1)^3 \).

Next we stretch this graph vertically by a factor of 2 to obtain the graph of \( y = -2(x+1)^3 \).

The last step is to shift this graph up 3 units to obtain the graph of \( y = -2(x+1)^3 + 3 \).
Chapter 2: Functions and Their Graphs

11. a. Graph of Set A:
Enter the x-values in L1 and the y-values in L2. Make a scatter diagram using STATPLOT and press [ZOOM] [9] to fit the data to your window.

Graph of Set B:
Enter the x-values in L1 and the y-values in L2. Make a scatter diagram using STATPLOT and press [ZOOM] [9] to fit the data to your window.

From the graphs, it appears that Set B is more linear. Set A has too much curvature.

b. Set B appeared to be the most linear so we will use that data set.
Press [STAT] [>] [4] [ENTER] to get the equation of the line (assuming the data is entered already).

The line of best fit for Set B is roughly $y = 2.02x - 5.33$.

b. The basic function is $y = |x|$ so we start with the graph of this function.

Next we shift this graph 4 units to the left to obtain the graph of $y = |x + 4|$.

Next we shift this graph up 2 units to obtain the graph of $y = |x + 4| + 2$.
12. a. \( r(x) = -0.115x^2 + 1.183x + 5.623 \)
For the years 1992 to 2004, we have values of \( x \) between 0 and 12. Therefore, we can let \( X_{\text{min}} = 0 \) and \( X_{\text{max}} = 12 \). Since \( r \) is the interest rate as a percent, we can try letting \( Y_{\text{min}} = 0 \) and \( Y_{\text{max}} = 10 \).

The highest rate during this period appears to be 8.67\%, occurring in 1997 \((x \approx 5)\).

b. For 2010, we have \( x = 2010 - 1992 = 18 \).

\[ r(18) = -0.115(18)^2 + 1.183(18) + 5.623 = -10.343 \]

The model predicts that the interest rate will be −10.343\%. This is not a reasonable value since it implies that the bank would be paying interest to the borrower.

13. a. Let \( x \) = width of the rink in feet. Then the length of the rectangular portion is given by \( 2x - 20 \). The radius of the semicircular portions is half the width, or \( r = \frac{x}{2} \).

To find the volume, we first find the area of the surface and multiply by the thickness of the ice. The two semicircles can be combined to form a complete circle, so the area is given by

\[ A = l \cdot w + \pi r^2 = (2x - 20)(x) + \pi \left(\frac{x}{2}\right)^2 = 2x^2 - 20x + \frac{\pi x^2}{4} \]

We have expressed our measures in feet so we need to convert the thickness to feet as well.

Cumulative Review 1-2

1. \(-5x + 4 = 0\)

\(-5x = -4\)

\(x = \frac{-4}{-5} = \frac{4}{5}\)

The solution set is \(\left\{\frac{4}{5}\right\}\).

3. \(3x^2 - 5x - 2 = 0\)

\((3x + 1)(x - 2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2\)

The solution set is \(\left\{-\frac{1}{3}, 2\right\}\).

5. \(4x^2 - 2x + 4 = 0 \Rightarrow 2x^2 - x + 2 = 0\)

\(x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)} = \frac{1 \pm \sqrt{-16}}{4} = \frac{1 \pm \sqrt{-16}}{4}\)

no real solution

7. \(\sqrt[5]{1-x} = 2\)

\((\sqrt[5]{1-x})^5 = 2^5\)

\(1-x = 32\)

\(-x = 31\)

\(x = -31\)

The solution set is \(\{-31\}\).
9. \[ 4x^2 - 2x + 4 = 0 \Rightarrow 2x^2 - x + 2 = 0 \]
   \[
   x = \frac{-(1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)} \\
   = \frac{1 \pm \sqrt{1 - 16}}{4} = \frac{1 \pm \sqrt{-15}}{4} = \frac{1 \pm i\sqrt{15}}{4}
   \]
   The solution set is \( \left\{ \frac{1 - \sqrt{15}i}{4}, \frac{1 + \sqrt{15}i}{4} \right\} \).

11. \[ -3x + 4y = 12 \Rightarrow 4y = 3x + 12 \]
   \[ y = \frac{3}{4}x + 3 \]
   This is a line with slope \( \frac{3}{4} \) and y-intercept (0, 3).

13. \[ x^2 + y^2 + 2x - 4y + 4 = 0 \]
   \[
   x^2 + 2x + y^2 - 4y = -4 \\
   (x^2 + 2x + 1) + (y^2 - 4y + 4) = -4 + 1 + 4 \\
   (x + 1)^2 + (y - 2)^2 = 1
   \]
   This is a circle with center \((-1, 2)\) and radius 1.

15. a. Domain: \( \{x \mid -4 \leq x \leq 4\} \)
   
   Range: \( \{y \mid -1 \leq y \leq 3\} \)
   
   b. Intercepts: \((-1, 0), (0, -1), (1, 0)\)
   
   x-intercepts: \(-1, 1\)
   
   y-intercept: \(-1\)

   c. The graph is symmetric with respect to the y-axis.

   d. When \( x = 2 \), the function takes on a value of 1. Therefore, \( f(2) = 1 \).

   e. The function takes on the value 3 at \( x = -4 \) and \( x = 4 \).

   f. \( f(x) < 0 \) means that the graph lies below the x-axis. This happens for \( x \) values between \(-1\) and 1. Thus, the solution set is \( \{x \mid -1 < x < 1\} \).

   g. The graph of \( y = f(x) + 2 \) is the graph of \( y = f(x) \) but shifted up 2 units.

   h. The graph of \( y = f(-x) \) is the graph of \( y = f(x) \) but reflected about the y-axis.

   i. The graph of \( y = 2f(x) \) is the graph of \( y = f(x) \) but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.

   j. Since the graph is symmetric about the y-axis, the function is even.
k. The function is increasing on the open interval \((0,4)\).

l. The function is decreasing on the open interval \((-4,0)\).

m. There is a local minimum of \(-1\) at \(x=0\). There are no local maxima.

n. \[
\frac{f(4)-f(1)}{4-1} = \frac{3-0}{3} = \frac{3}{3} = 1
\]
The average rate of change of the function from 1 to 4 is 1.

17. \(y = x^3 - 3x + 1\)

(a) \((-2,-1)\)

\((-2)^3 - (3)(-2) + 1 = -8 + 6 + 1 = -1\)

\((-2,-1)\) is on the graph.

(b) \((2,3)\)

\((2)^3 - (3)(2) + 1 = 8 - 6 + 1 = 3\)

\((2,3)\) is on the graph.

(c) \((3,1)\)

\((3)^3 - (3)(3) + 1 = 27 - 9 + 1 = 19 \neq 1\)

\((3,1)\) is not on the graph.

19. Use ZERO (or ROOT) on the graph of \(y_1 = x^4 - 3x^3 + 4x - 1\).

21. Yes, each \(x\) corresponds to exactly 1 \(y\).

23. \(h(z) = \frac{3z-1}{z^2 - 6z - 7}\)

The denominator cannot be zero:

\[z^2 - 6z - 7 \neq 0\]

\((z+1)(z-7) \neq 0\)

\(z \neq -1\) or \(7\)

Domain: \(\{z | z \neq -1, z \neq 7\}\)

25. \(f(x) = \frac{x}{x+4}\)

a. \(f(\frac{1}{4}) = \frac{1}{\frac{1}{4} + 4} = \frac{1}{5} \neq \frac{1}{4}\)

\(\left(\frac{1}{4}\right)\) is not on the graph of \(f\)

b. \(f(-2) = \frac{-2}{-2+4} = \frac{-2}{2} = -1\)

\((-2,-1)\) is on the graph of \(f\)

c. Solve for \(x:\)

\[
\frac{x}{x+4} = 2
\]

\[
x = 2(x+4)
\]

\[
x = 2x + 8
\]

\[
-8 = x
\]

\((-8,2)\) is on the graph of \(f\).