Connected Mathematics is committed to the development of mathematical skills—skills that are much more than just quickness with paper-and-pencil algorithms. The overarching goal of CMP discussed on page 2 makes this commitment to skill:

All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics, including the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.

In Connected Mathematics, students develop understanding of algorithms and strategies for computing and estimating in a variety of ways. They learn to recognize when an algorithm or strategy applies to a new context and when they can build on the skills and strategies they know in order to develop new strategies. In these processes, students practice skills as an ongoing activity throughout the curriculum.

Students need to know how and when to use paper-and-pencil algorithms, mental computation, calculator procedures, and estimation strategies. They need to recognize when an exact answer is required and when an approximate answer is sufficient, and they need a variety of methods for finding an answer. In some situations an approximate answer is sufficient and in these situations a paper-and-pencil algorithm may not be the most efficient (or practical) method. In Bits and Pieces III Problem 4.2, Question D(1) students estimate to find a 20% tip for $7.93. It is more efficient for most students to estimate that 10% is about 80 cents so 20% is $1.60 than to multiply 0.20 times 7.93 to obtain $1.57.

Students also need to know methods for judging the reasonableness of an answer. For example, to estimate or judge the reasonableness of the answer to the sum $\frac{2}{3}$ and $\frac{1}{3}$, students might argue that $\frac{2}{3}$ is close to but less than $\frac{1}{2}$, and $\frac{1}{3}$ is more than $\frac{1}{4}$ but less than $\frac{1}{2}$. Thus, the answer is more than $\frac{1}{2}$ but less than 1, or about $\frac{3}{4}$.

Skills with the four basic operations on fractions are developed and maintained throughout the curriculum. Students should be able to add two simple fractions quickly by finding a common denominator, but they should also understand why this algorithm works. Connected Mathematics helps students build a strong foundation for the development of addition, subtraction, multiplication, and division of fractions. In this phase, the essential building blocks of equivalent fractions, meaning of fractions, models and representation of fractions are developed and used. For example, in Bits and Pieces I, students develop an understanding of equivalences. In Problem 2.2 students represent fractions on a number line and use the number line to develop a method of finding equivalent fractions. A portion of this problem is shown on the next page.

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**C.** The sales tax in Kadisha’s state is 5%. Kadisha says she computes a 15% tip by multiplying the tax shown on her bill by three. For a bill with a tax charge of $0.38, Kadisha’s tip is $0.38 \times 3 = $1.14.

1. Why does Kadisha’s method work?
2. Use a similar method to compute a 20% tip. Explain.

**D.** When people leave a 15% or 20% tip, they often round up to the nearest multiple of 5 or 10 cents. For example, in Question C, Kadisha might leave a tip of $1.15 rather than $1.14.

1. If Kadisha always rounds up, what is a 20% tip on her bill?

<table>
<thead>
<tr>
<th>ITEM</th>
<th>AMOUNT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>$7.55</td>
</tr>
<tr>
<td>Tax</td>
<td>$0.38</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$7.93</td>
</tr>
</tbody>
</table>

---

**Bits and Pieces III • page 53**
### Bits and Pieces I • page 22

**Problem 2 Finding Equivalent Fractions**

A. 1. On a number line like the one below, carefully label marks that show where $\frac{4}{2}$ and $\frac{2}{4}$ are located.

```
\[ \begin{array}{c}
\hline
0 & \quad \frac{1}{2} & \quad 1 \\
\hline
\end{array} \]
```

2. Use the same number line. Mark the point that is halfway between 0 and $\frac{1}{2}$ and the point that is halfway between $\frac{1}{2}$ and 1.

3. Label these new marks with appropriate fraction names.

4. What are additional ways to label $\frac{1}{4}$ and $\frac{3}{4}$? Explain.

5. Use the same number line. Mark halfway between each of the marks that were already made.

6. Label the new marks on your number line. Add additional names to the marks that were already named.

7. Write three number sentences that show equivalent fractions on your number line. (Here is an example: $\frac{1}{2} = \frac{2}{4}$)

8. Write two number sentences to show fractions that are equivalent to $\frac{1}{2}$.

B. 1. On your number line, the distance between the $\frac{1}{4}$ mark and the $\frac{1}{2}$ mark is $\frac{1}{4}$ of a unit. The distance between the $\frac{1}{2}$ mark and the 1 mark is $\frac{1}{2}$ of a unit. Name two other fractions that are $\frac{1}{4}$ of a unit apart on your number line.

2. What is the distance between the $\frac{1}{4}$ and $\frac{1}{2}$ marks on your number line? How do you know?

3. Name at least two other fraction pairs that are the same distance apart as $\frac{1}{4}$ and $\frac{1}{2}$.

4. Describe the distance between $\frac{1}{4}$ and $\frac{1}{2}$ in two ways.

**Example 1**

**Using Fraction Multiplication Skills**

In *Bits and Pieces III*, the algorithm of multiplying decimals is connected to multiplying fractions. Here students use their fraction multiplication skills as a strategy to multiply decimals.

### Bits and Pieces III • page 22

#### Getting Ready for Problem 2

To find the product of $0.3 \times 2.3$, you can use equivalent fractions.

$$0.3 = \frac{3}{10} \quad \text{and} \quad 2.3 = \frac{23}{10} \Rightarrow 0.3 \times 2.3 = \frac{3}{10} \times \frac{23}{10}$$

* What is the product written as a fraction?

* What is the product written as a decimal?

* How can knowing the product as a fraction help you write the product in decimal form?

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### Bits and Pieces III • page 22

**Example 2**

**Using Fraction Multiplication Skills**

Students continue to use their understanding of equivalent fractions to build and consolidate their understanding of new ideas in new contexts. The sequence of the units is carefully chosen with this in mind. For example, in *Stretching and Shrinking*, students use fractional scale factors to find and identify similar figures.

A portion of an Application question and a solution are given below to illustrate continued use of fraction multiplication.

#### Stretching and Shrinking • page 86

For Exercises 23 and 24 on page 87, use the rectangles below. The rectangles are not shown at actual size.

![Image](image-url)

**Multiple Choice**

23. Which pair of rectangles is similar?

A. L and M  
B. L and Q  
C. L and N  
D. P and R

24. a. Find at least one more pair of similar rectangles.

b. For each similar pair, find both the scale factor relating the larger rectangle to the smaller rectangle and the scale factor relating the smaller rectangle to the larger rectangle.

c. For each similar pair, find the ratio of the area of the larger rectangle to the area of the smaller rectangle.

**Possible Solution**

One pair of similar rectangles is Rectangles M and Q. The scale factor from Rectangle Q to Rectangle M is $\frac{2}{3}$. This means that scaling the dimensions of Rectangle Q by $\frac{2}{3}$ results in the dimensions of Rectangle M.

Use fraction multiplication to verify that this is the correct scale factor: $3 \text{ cm} \times \frac{2}{3} = 2 \text{ cm}$.

As illustrated with rational numbers above, a similar development is given to integers in *Accentuate the Negative* and irrational numbers in *Looking for Pythagoras*. As students move through the curriculum, they expand their work with the real number system and continue to practice operating with real numbers in a variety of situations.
Proportional reasoning skills are essential to a student’s mathematical development. Many problems in K–12 mathematics and beyond call for students to utilize proportional reasoning skills. It is the core idea in being able to write equivalent fractions, in all operations with fractions, in making sense of scales, in similarity, in size transformations, and in solving some linear equations. Students can learn to mimic these skills by learning a new strategy for every skill, but their learning will be much more powerful if they can see an underlying idea and its connections. In the following problem from Comparing and Scaling, students are developing proportional reasoning skills. As they solve Problem 2.1, they build on their prior knowledge of fractions and rational numbers.

### Problem 2.1: Mixing Juice

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times. One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange-juice concentrate. To find the mix that tastes best, they decide to test some mixes.

#### Developing Comparison Strategies

A. Which mix will make juice that is the most “orangey”? Explain.
B. Which mix will make juice that is the least “orangey”? Explain.
C. Which comparison statement is correct? Explain.
D. Assume that each camper will get \( \frac{1}{2} \) cup of juice.

1. For each mix, how many batches are needed to make juice for 240 campers?
2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?
E. For each mix, how much concentrate and how much water are needed to make 1 cup of juice?

#### Team 1 Student Work

<table>
<thead>
<tr>
<th>Mix A</th>
<th>Mix B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cups concentrate</td>
<td>5 cups concentrate</td>
</tr>
<tr>
<td>3 cups cold water</td>
<td>9 cups cold water</td>
</tr>
</tbody>
</table>

#### Team 3 Student Work

<table>
<thead>
<tr>
<th>Mix A</th>
<th>Mix B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{3} ) concentrate ( \frac{1}{1.5} ) water</td>
<td>( \frac{5}{9} ) concentrate ( \frac{1}{1.2} ) water</td>
</tr>
</tbody>
</table>

The students of Team 1 used a part-to-whole strategy. They showed that they knew how to find and use percents to make comparisons.

The students on Team 3 used a part-to-part strategy. They gave the ratio in fraction form and then made the numerators the same to make comparisons. They recognized that the smallest denominator shows the mix that is the most “orangey,” because it uses the least water per can of concentrate. The work shows that students have flexibility in using fractions, decimals, and percents to make their comparisons of ratios.
Knowing when to use a particular operation is also a skill. *Bits and Pieces III* is designed to provide experiences in building algorithms for the four basic operations with decimals, as well as opportunities for students to consider when such operations are useful in solving problems. For example, what features of a problem indicate to the student that division will help solve it? Building this kind of thinking and reasoning supports the development of skill with the algorithms. In Problem 3.1 B. of *Bits and Pieces III*, students need to interpret the problem situation and determine what decimal operation will solve the problem. This skill is practiced as students begin to develop and use algorithms for decimal operations.

*Connected Mathematics* recognizes that students must have an opportunity to practice skills in a variety of situations throughout the course of their mathematical career. The Connections feature of the ACE Exercises (discussed more on page 43) offers a way for student to continue practicing skills learned in previous units. As a problem-centered curriculum, *Connected Mathematics* provides students the opportunity to use their skills in a wide range of situations that promote higher-order thinking and help students develop problem-solving skills essential to their mathematical future in school and in life. Students explore both problem situations that are purely mathematical and others that are real world. They learn to use what they know to solve contextualized situations and to do computation in “naked” number situations.

**Bits and Pieces III • page 37**

- Examine each situation. Decide what operation to use and then estimate the size of the answer.

1. Ashley eats five 5.25-ounce slices of watermelon in a contest at the picnic. How many ounces of watermelon does she eat?

2. Stacey needs $39.99 for a pair of sneakers. She has $22.53 in her savings and a $15 check from babysitting. Can she buy the shoes?

3. Li Ming’s allowance for transportation is $12.45. How many times can she ride the bus if it costs $0.75 a trip?