The Connected Mathematics Project (CMP) was funded by the National Science Foundation between 1991 and 1997 to develop a mathematics curriculum for grades 6, 7, and 8. The result was Connected Mathematics, a complete mathematics curriculum that helps students develop understanding of important concepts, skills, procedures, and ways of thinking and reasoning in number, geometry, measurement, algebra, probability, and statistics.

In 2000, the National Science Foundation funded a revision of the Connected Mathematics materials, CMP2, to take advantage of what we learned in the six years that the first edition of CMP has been used in schools. This Implementation Guide elaborates the goals of CMP2, the process we used for the revision, the scope of the curriculum, and a process for implementation that will support student and teacher learning.

### CMP: A Curriculum for Students and Teachers

The CMP materials reflect the understanding that teaching and learning are not distinct—“what to teach” and “how to teach it” are inextricably linked. The circumstances in which students learn affect what is learned. The needs of both students and teachers are considered in the development of the CMP curriculum materials. This curriculum helps teachers and those who work to support teachers examine their expectations for students and analyze the extent to which classroom mathematics tasks and teaching practices align with their goals and expectations.

### Overarching Goal of CMP

The overarching goal of Connected Mathematics is to help students and teachers develop mathematical knowledge, understanding, and skill along with an awareness of and appreciation for the rich connections among mathematical strands and between mathematics and other disciplines. All the CMP curriculum development has been guided by a single mathematical standard.

*All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics, including the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.*
**CMP2 at a Glance**

Below are some key features of *Connected Mathematics 2*:

**Problem Centered**
Important mathematical concepts are embedded in engaging problems. Students develop understanding and skill as they explore the problems, individually, in a group, or with the class.

**Practice With Concepts and Related Skills**
The in-class development problems and the homework exercises give students practice distributed over time with important concepts, related skills, and algorithms.

**Complete Curriculum**
The twenty-four *Connected Mathematics 2* units—eight units for each grade—form a complete middle school curriculum that develops mathematical skills and conceptual understanding across mathematical strands. (Three units from the first edition of CMP—*Ruins of Montarek*, *Data Around Us*, and *Clever Counting*—will continue to be available to help schools reach individual state mathematics expectations.) In addition, the program provides a complete assessment package, including quizzes, tests, and projects.

**For Teachers as well as Students**
The *Connected Mathematics* materials were written to support teacher learning of both unfamiliar content and pedagogical strategies. The Teacher’s Guides include extensive help with mathematics, pedagogy, and assessment.

**Research Based**
Each *Connected Mathematics* unit was field tested, evaluated, and revised over a five-year period. Approximately 200 teachers and 45,000 students in diverse school settings across the United States participated in the development of the curriculum.

**It Works**
It works. Research results consistently show CMP students outperform non-CMP students on tests of problem-solving ability, conceptual understanding, and proportional reasoning. And CMP students do as well as, or better than, non-CMP students on tests of basic skills.
Influence of Theory and Research on CMP2 Curriculum

The curriculum, teacher support, and assessment materials that comprise the Connected Mathematics program reflect influence from a variety of sources:

• knowledge of theory and research;
• authors’ imagination and personal teaching and learning experiences;
• advice from teachers, mathematicians, teacher educators, curriculum developers, and mathematics education researchers;
• advice from teachers and students who used pilot and field-test versions of the materials.

The fundamental features of the CMP program—focus on big ideas of middle grades mathematics, teaching through student-centered exploration of mathematically rich problems, and continual assessment to inform instruction—reflect the distillation of advice and experience from those varied sources.

Our work was influenced in significant ways by what we knew of existing theory and research in mathematics education. Here we mention and explain briefly the key themes in the theory/research basis for our work.

Research From the Cognitive Sciences

1. Social Constructivism We are in general agreement with constructivist explanations of the ways that knowledge is developed, especially the social constructivist ideas about influence of discourse on learning. This position is reflected in the authors’ decision to write materials that would support student-centered investigation of mathematical problems and in our attempt to design problem content and formats that would encourage student-student and student-teacher dialogue about the work.

2. Conceptual and Procedural Knowledge We have been influenced by theory and research indicating that mathematical understanding is fundamentally a web of logical and psychological connections among ideas. Furthermore, we have interpreted research on the interplay of conceptual and procedural knowledge to say that sound conceptual understanding is an important foundation for procedural skill, not an incidental and delayed consequence of repeated rote procedural practice.

3. Multiple Representations An important indication of students’ connected mathematical knowledge is their ability to represent ideas in a variety of ways. We have interpreted this theory to imply that curriculum materials should frequently provide and ask for knowledge representation using graphs, number patterns, written explanations, and symbolic expressions.

4. Cooperative Learning There is a consistent and growing body of research indicating that when students engage in cooperative work on appropriate problem-solving tasks, their mathematical and social learning will be enhanced. We have interpreted this line of theory and research to imply that we should design student and teacher materials that are suitable for use in cooperative learning instructional formats as well as individual learning formats—the mathematical tasks dictate the format.

Research From Mathematics Education

5. Rational Numbers/Proportional Reasoning The extensive psychological literature on development of rational numbers and proportional reasoning has guided our development of curriculum materials addressing this important middle school topic. Furthermore, the implementation of CMP materials in real classrooms has allowed us to contribute to that literature with research
publications that show the effects of new teaching approaches to traditionally difficult topics.

6. **Probability and Statistical Reasoning**
The interesting research literature concerning development of and cognitive obstacles to student learning of statistical concepts, such as mean and graphic displays, and probability concepts, such as the law of large numbers and conditional probability, has been used as we developed the statistics and probability units of CMP materials.

7. **Algebraic Reasoning**
The different conceptualizations of algebra described and researched in the literature contributed to the treatment of algebra in CMP. Various scholars describe algebra as a study of modeling, functions, generalized arithmetic, and/or as a problem-solving tool. CMP has aspects of each of these descriptions of algebra, but focuses more directly on functions and on the effects of rates of change on representations. The research literature illuminates some of the cognitive complexities inherent in algebraic reasoning and offers suggestions on helping students overcome difficulties. Research concerning concepts, such as equivalence, functions, the equal sign, algebraic variables, graphical representations, multiple representations, and the role of technology, were used as we developed the algebra units of the CMP materials.

8. **Geometric/Measurement Reasoning**
Results from national assessments and research findings show that student achievement in geometry and measurement is weak. Research on student understanding and learning of geometric/measurement concepts, such as angle, area, perimeter, volume, and processes such as visualization, contributed to the development of geometry/measurement units in CMP materials. As a result of research shifting from a focus on shape and form to the related ideas of congruence, similarity, and symmetry transformations, CMP geometry units were designed to focus on these important ideas.

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**Research From Education Policy and Organization**

9. **Motivation**
One of the fundamental challenges in mathematics teaching is convincing students that serious effort in study of the subject will be rewarding and that learning of mathematics can also be an enjoyable experience. We have paid careful attention to literature on extrinsic and intrinsic motivation, and we have done some informal developmental research of our own to discover aspects of mathematics and teaching that are most effective in engaging student attention and interest.

10. **Teacher and School Change**
The most attractive school mathematics curriculum materials will be of little long-term value or effect if they are not put into use in schools. In the process of helping teachers through professional development, we have paid close attention to what is known about effective teacher professional development and the school strategies that seem to be most effective.


While each of these ten points indicates influence of theory and research on design and development of the CMP curriculum, teacher, and assessment materials, it would be misleading to suggest that the influence is direct and controlling in all decisions. As the authors have read the research literature reporting empirical and theoretical work, research findings and new ideas have been absorbed and factored into the creative, deliberative, and experimental process that leads to a comprehensive mathematics program for schools.
The authors were guided by the following principles in the development of the *Connected Mathematics* materials. These statements reflect both research and policy stances in mathematics education about what works to support students’ learning of important mathematics.

- The “big” or key mathematical ideas around which the curriculum is built are identified.
- The underlying concepts, skills, or procedures supporting the development of a key idea are identified and included in an appropriate development sequence.
- An effective curriculum has coherence—it builds and connects from investigation to investigation, unit-to-unit, and grade-to-grade.
- Classroom instruction focuses on inquiry and investigation of mathematical ideas embedded in rich problem situations.
- Mathematical tasks for students in class and in homework are the primary vehicle for student engagement with the mathematical concepts to be learned. The key mathematical goals are elaborated, exemplified, and connected through the problems in an investigation.
- Ideas are explored through these tasks in the depth necessary to allow students to make sense of them. Superficial treatment of an idea produces shallow and short-lived understanding and does not support making connections among ideas.
- The curriculum helps students grow in their ability to reason effectively with information represented in graphic, numeric, symbolic, and verbal forms and to move flexibly among these representations.
- The curriculum reflects the information-processing capabilities of calculators and computers and the fundamental changes such tools are making in the way people learn mathematics and apply their knowledge of problem-solving tasks.

*Connected Mathematics* is different from many more familiar curricula in that it is problem centered. The following section elaborates what we mean by this and what the value added is for students of such a curriculum.

**Rationale for a Problem-Centered Curriculum**

Students’ perceptions about a discipline come from the tasks or problems with which they are asked to engage. For example, if students in a geometry course are asked to memorize definitions, they think geometry is about memorizing definitions. If students spend a majority of their mathematics time practicing paper-and-pencil computations, they come to believe that mathematics is about calculating answers to arithmetic problems as quickly as possible. They may become faster at performing specific types of computations, but they may not be able to apply these skills to other situations or to recognize problems that call for these skills.

Formal mathematics begins with undefined terms, axioms, and definitions and deduces important conclusions logically from those starting points. However, mathematics is produced and used in a much more complex combination of exploration, experience-based intuition, and reflection. If the purpose of studying mathematics is to be able to solve a variety of problems, then students need to spend significant portions of their mathematics time solving problems that require thinking, planning, reasoning, computing, and evaluating.

A growing body of evidence from the cognitive sciences supports the theory that students can make sense of mathematics if the concepts and skills are embedded within a context or problem. If time is spent exploring interesting mathematics situations, reflecting on solution methods, examining why the methods work, comparing methods, and relating methods to those used in previous situations, then students are likely to build more robust understanding of mathematical concepts and related procedures. This method is quite different from the assumption that students learn by observing a teacher as he or she demonstrates how to solve a problem and then practices that method on similar problems.
A problem-centered curriculum not only helps students to make sense of the mathematics, it also helps them to process the mathematics in a retrievable way.

Teachers of CMP report that students in succeeding grades remember and refer to a concept, technique, or problem-solving strategy by the name of the problem in which they encountered the ideas. For example, the Basketball Problem from *What Do You Expect?* in Grade Seven becomes a trigger for remembering the processes of finding compound probabilities and expected values.

Results from the cognitive sciences also suggest that learning is enhanced if it is connected to prior knowledge and is more likely to be retained and applied to future learning. Critically examining, refining, and extending conjectures and strategies are also important aspects of becoming reflective learners.

In CMP, important mathematical ideas are embedded in the context of interesting problems. As students explore a series of connected problems, they develop understanding of the embedded ideas and, with the aid of the teacher, abstract powerful mathematical ideas, problem-solving strategies, and ways of thinking. They learn mathematics and learn how to learn mathematics.

### Characteristics of Good Problems

To be effective, problems must embody critical concepts and skills and have the potential to engage students in making sense of mathematics. And, since students build understanding by reflecting, connecting, and communicating, the problems need to encourage them to use these processes.

Each problem in *Connected Mathematics* satisfies the following criteria:

- The problem must have important, useful mathematics embedded in it.
- Investigation of the problem should contribute to students’ conceptual development of important mathematical ideas.
- Work on the problem should promote skillful use of mathematics and opportunities to practice important skills.
- The problem should create opportunities for teachers to assess what students are learning and where they are experiencing difficulty.

In addition each problem satisfies some or all of the following criteria:

- The problem should engage students and encourage classroom discourse.
- The problem should allow various solution strategies or lead to alternative decisions that can be taken and defended.
- Solution of the problem should require higher-level thinking and problem solving.
- The mathematical content of the problem should connect to other important mathematical ideas.

### Practice With Concepts, Related Skills, and Algorithms

Students need to practice mathematical concepts, ideas, and procedures to reach a level of fluency that allows them to “think” with the ideas in new situations. To accomplish this we were guided by the following principles related to skills practice.

- Immediate practice should be related to the situations in which the ideas have been developed and learned.
- Continued practice should use skills and procedures in situations that connect to ideas that students have already encountered.
- Students need opportunities to use the ideas and skills in situations that extend beyond familiar situations. These opportunities allow students to use skills and concepts in new combinations to solve new kinds of problems.
- Students need practice distributed over time to allow high ideas, concepts and procedures to reach a level of fluency of use in familiar and unfamiliar situations and to build connections to other concepts and procedures.
- Students need guidance in reflecting on what they are learning, how the ideas fit together, and how to make judgments about what is helpful in which kinds of situations.
- Throughout the Number and Algebra Strands development, students need to learn how to make judgments about what operation or combination of operations or representations is useful in a given situation, as well as, how to become skillful at carrying out the needed computation(s). Knowing how to, but not when to, is insufficient.
Rationale for Depth versus Spiraling

The concept of a “spiraling” curriculum is philosophically appealing; but, too often, not enough time is spent initially with a new concept to build on it at the next stage of the spiral. This leads to teachers spending a great deal of time re-teaching the same ideas over and over again. Without a deeper understanding of concepts and how they are connected, students come to view mathematics as a collection of different techniques and algorithms to be memorized.

Problem solving based on such learning becomes a search for the correct algorithm rather than seeking to make sense of the situation, considering the nature and size of a solution, putting together a solution path that makes sense, and examining the solution in light of the original question. Taking time to allow the ideas studied to be more carefully developed means that when these ideas are met in future units, students have a solid foundation on which to build. Rather than being caught in a cycle of relearning the same ideas superficially which are quickly forgotten, students are able to connect new ideas to previously learned ideas and make substantive advances in knowledge.

With any important mathematical concept, there are many related ideas, procedures, and skills. At each grade level, a small, select set of important mathematical concepts, ideas, and related procedures are studied in depth rather than skimming through a larger set of ideas in a shallow manner. This means that time is allocated to develop understanding of key ideas in contrast to “covering” a book. The Teacher’s Guides accompanying CMP materials were developed to support teachers in planning for and teaching a problem-centered curriculum. Practice on related skills and algorithms are provided in a distributed fashion so that students not only practice these skills and algorithms to reach facility in carrying out computations, but they also learn to put their growing body of skills together to solve new problems.

Developing Depth of Understanding and Use

Through the field trials process we were able to develop units that result in student understanding of key ideas in depth. An example is illustrated in the way that Connected Mathematics treats proportional reasoning—a fundamentally important topic for middle school mathematics and beyond. Conventional treatments of this central topic are often limited to a brief expository presentation of the ideas of ratio and proportion, followed by training in techniques for solving proportions. In contrast, the CMP curriculum materials develop core elements of proportional reasoning in a seventh grade unit, Comparing and Scaling, with the groundwork for this unit having been developed in four prior units. Five succeeding units build on and connect to students’ understanding of proportional reasoning. These units and their connections are summarized as follows:

Grade 6 Bits and Pieces I and II introduce students to fractions and their various meanings and uses. Models for making sense of fraction meanings and of operating with fractions are introduced and used. These early experiences include fractions as ratios. The extensive work with equivalent forms of fractions builds the skills needed to work with ratio and proportion problems. These ideas are developed further in the probability unit How Likely Is It? in which ratio comparisons are informally used to compare probabilities. For example, is the probability of drawing a green block from a bag the same if we have 10 green and 15 red or 20 green and 30 red?

Grade 7 Stretching and Shrinking introduces proportionality concepts in the context of geometric problems involving similarity. Students connect visual ideas of enlarging and reducing figures, numerical ideas of scale factors and ratios, and applications of similarity through work with problems focused around the question: “What would it mean to say two figures are similar?”

The next unit in grade seven is the core proportional reasoning unit, Comparing and Scaling, which connects fractions, percents, and ratios through investigation of various situations in which the central question is: “What strategies make sense in describing how much greater one quantity is than another?” Through a series of
problem-based investigations, students explore the meaning of ratio comparison and develop, in a progression from intuition to articulate procedures, a variety of techniques for dealing with such questions.

A seventh grade unit that follows, *Moving Straight Ahead*, is a unit on linear relationships and equations. Proportional thinking is connected and extended to the core ideas of linearity—constant rate of change and slope. Then in the probability unit *What Do You Expect?*, students again use ratios to make comparisons of probabilities.

**Grade 8 Thinking With Mathematical Models; Looking For Pythagoras; Growing, Growing, Growing, and Frogs, Fleas, and Painted Cubes** extend the understanding of proportional relationships by investigating the contrast between linear relationships and inverse, exponential, and quadratic relationships. Also in Grade Eight, *Samples and Populations* uses proportional reasoning in comparing data situations and in choosing samples from populations.

These unit descriptions show two things about *Connected Mathematics*—the in-depth development of fundamental ideas and the connected use of these important ideas throughout the rest of the units.

**Support for Classroom Teachers**

When mathematical ideas are embedded in problem-based investigations of rich context, the teacher has a critical responsibility for ensuring that students abstract and generalize the important mathematical concepts and procedures from the experiences with the problems. In a problem-centered classroom, teachers take on new roles—moving from always being the one who does the mathematics to being the one who guides, interrogates, and facilitates the learner in doing and making sense of the mathematics.

The Teacher’s Guides and Assessment Resources developed for *Connected Mathematics* provide these kinds of help for the teacher:

- The Teacher’s Guide for each unit engages teachers in a conversation about what is possible in the classroom around a particular lesson. Goals for each lesson are articulated. Suggestions are made about how to engage the students in the mathematics task, how to promote student thinking and reasoning during the exploration of the problem, and how to summarize with the students the important mathematics embedded in the problem. Support for this Launch—Explore—Summarize sequence occurs for each problem in the CMP curriculum.

- An overview and elaboration of the mathematics of the unit is located at the beginning of each Teacher’s Guide, along with examples and a rationale for the models and procedures used. This mathematical essay helps a teacher stand above the unit and see the mathematics from a perspective that includes the particular unit, connects to earlier units, and projects to where the mathematics goes in subsequent units and years.

- Actual classroom scenarios are included to help stimulate teachers’ imaginations about what is possible.

- Questions to ask students at all stages of the lesson are included to help teachers support student learning.

- Reflection questions are provided at the end of each investigation to help teachers assess what sense students are making of the ‘big’ ideas and to help students abstract, generalize, and record the mathematical ideas and techniques developed in the Investigation.

- Diverse kinds of assessments are included in the student units and the Assessment Resources that mirror classroom practices as well as highlight important concepts, skills, techniques, and problem solving strategies.

- Multiple kinds of assessment are included to help teachers see assessment and evaluation as a way to inform students of their progress, apprise parents of students’ progress, and guide the decisions a teacher makes about lesson plans and classroom interactions.

See pages 73–77 for more details about teacher support materials.
Research, Field Testing, and Evaluation

Before starting the design phase of the materials, we commissioned individual reviews of the first edition of CMP units from 84 individuals in 17 states and comprehensive reviews from more than 20 schools in 14 states.

Individual Reviews These reviews focused on particular strands over all three grades (such as number, algebra, or statistics) on particular sub-populations (such as students with special needs or students who are commonly underserved), or on topical concerns (such as language use and readability).

Comprehensive Reviews These reviews were conducted in groups that included teachers, administrators, curriculum supervisors, mathematicians, experts in special education, language, and reading level analyses, English language learners, issues of equity, and others. Each group reviewed an entire grade level of the curriculum. All responses were coded and entered into a database that allowed reports to be printed for any issue or combination of issues that would be helpful to an author or staff person in designing a unit.

In addition, CMP issued a call to schools to serve as pilot schools for the development of CMP2. We received 50 applications from districts for piloting. From these applications we chose 15 that included 49 school sites in 12 states and the District of Columbia. We received evaluation feedback from these sites over the five-year cycle of development.

Based on the commissioned reviews, what the authors had learned from CMP schools over a 6-year period, and input from our Advisory Board, the authors started with grades 6 and 7 and systematically revised and restructured the units and their sequence for each grade-level to create a first draft of the revision. These were sent to our pilot schools to be taught during the second year of the project. These initial grade level unit drafts were the basis for substantial feedback from our trial teachers.

Examples of the kinds of questions we asked the trial teachers following each iteration of a unit or grade level are given below.

**“BIG PICTURE” UNIT FEEDBACK**

1. Is the mathematics of the unit important for students at this grade level? Explain.
2. Are the mathematical goals of the unit clear to you?
3. Overall, what are the strengths and weaknesses in this unit?
4. Please comment on your students’ achievement of mathematics understanding at the end of this unit. What concepts/skills did they “nail”? Which concepts/skills are still developing? Which concepts/skills need a great deal more reinforcement?
5. Is there a flow to the sequencing of the Investigations? Does the mathematics develop smoothly throughout the unit? Are there any big leaps where another problem is needed to help students understand a big idea in an Investigation? What adjustments did you make in these rough spots?

**PROBLEM-BY-PROBLEM FEEDBACK**

1. Are the mathematical goals of each problem/investigation clear to you?
2. Is the language and wording of each problem understandable to students?
3. Are there any grammatical or mathematical errors in the problems? (Please be as specific as possible.)
4. Are there any problems that you think can be deleted?
5. Are there any problems that needed serious revision?

**APPLICATIONS•CONNECTIONS•EXTENSIONS FEEDBACK**

1. Does the format of the ACE exercises work for you and your students? Why or why not?
2. Which ACE exercises work well, which would you change, and why?
3. What needs to be added to or deleted from the ACE exercises? Is there enough practice for
4. Are there sufficient ACE exercises that challenge your more interested and capable students? If not, what needs to be added and why?

5. Are there sufficient ACE exercises that are accessible to and helpful to students that need more scaffolding for the mathematical ideas?

MATHEMATICAL REFLECTIONS AND LOOKING BACK, LOOKING AHEAD FEEDBACK

1. Are these reflections useful to you and your students in identifying and making more explicit the “big” mathematical ideas in the unit? If not, how could they be improved?

ASSESSMENT MATERIALS FEEDBACK

1. Are the check-ups, quizzes, unit tests, and projects useful to you? If not, how can they be improved? What should be deleted and what should be added? (Please give specifics.)

2. How do you use the assessment materials? Do you supplement the materials? If so, how and why?

TEACHER’S GUIDE FEEDBACK

1. Is the Teacher’s Guide useful to you? If not, what changes do you suggest and why?

2. Which parts of the Teacher’s Guide help you and which do you ignore or seldom use?

3. What would be helpful to add or expand in the Teacher’s Guide?

YEAR-END GRADE LEVEL FEEDBACK

1. Are the mathematical concepts, skills and processes in the units appropriate for the grade level?

2. Is the grade level placement of units optimal for your school district? Why or why not?

3. Does the mathematics flow smoothly for the students over the year?

4. Once an idea is learned, is there sufficient reinforcement and use in succeeding units?

5. Are connections made between units within the grade level?

6. Does the grade level sequence of units seem appropriate? If not, what changes would you make and why?

7. Overall, what are the strengths and weaknesses in the units for the year? (Please be as specific as possible.)

BIG PICTURE QUESTION

1. What three to five things would you have us seriously improve, change, or drop at each grade level? Please be specific about exactly what you suggest and why you would like to see this change.

Development Summary

CMP development followed the very rigorous design, field-test, evaluate loop pictured in the diagram below.

The units for each grade level went through at least three cycles of field trials–data feedback–revision. If needed, units had four rounds of field trials. This process of (1) commissioning reviews from experts, (2) using the field trials–feedback loops for the materials, (3) conducting key classroom observations by the CMP staff of units being taught, and (4) monitoring student performance on state and local tests by trial schools comprises research-based development of curriculum. This process takes five years to produce the final drafts of units that are sent to the publisher. Another 6 to 18 months is needed for editing, design, and layout for the published units. This process produces materials that are cohesive and effectively sequenced.
An Example of Effective Sequencing of Problems

To be effective, problems must be carefully sequenced to help students develop appropriate understanding and skill. The following set of problems from the Grade 6 unit, *Covering and Surrounding*, develops methods for finding the circumference and area of a circle.

The first problem uses the context of irregularly-shaped lakes to explore possible relationships between the perimeter of a curved figure and its area. Using a square grid to estimate perimeter and area helps students to understand the meaning of perimeter and area before using formulas.

In the second problem, students measure the diameter and circumference of several circular objects. They create a table and a graph of their data and look for a pattern relating the two measurements. Students should discover that the circumference is the diameter times “a little bit more than 3.” With the help of the teacher, students are introduced to the idea of pi or $\pi$ and find a closer approximation of its value.

The third problem asks students to estimate the area of a circle. Students are encouraged to think of several different methods and to explain their thinking. This problem is intended to motivate the need for a shortcut for calculating the area. To answer Parts C and D, students must consider the relationships between each of the measurements—radius, diameter, circumference, and area—and the possible price of each pizza.

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Covering and Surrounding • page 71

**Problem 5.1 Estimating Perimeter and Area**

Scale pictures for Loon Lake and Ghost Lake are on the grid.

A. Estimate the area and perimeter of Loon Lake and Ghost Lake.
B. Which lake is larger? Explain your reasoning.
C. Use your estimates to answer the questions. Explain your answers
   1. Naturalists claim that water birds need long shorelines for nesting and fishing. Which lake will better support water birds?
   2. Sailboaters and waterskiers want a lake with room to cruise. Which lake works better for boating and skiing?
   3. Which lake has more space for lakeside campsites?
   4. Which lake is better for swimming, boating, and fishing? Which lake is better for the nature preserve?
D. Is your estimate of the area of each lake more or less than the actual area of that lake? Explain. How could you get a more accurate estimate?

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Covering and Surrounding • page 73

**Problem 5.2 Finding Circumference**

When you want to find out if measurements are related, looking at patterns from many examples will help. Record your results in a table with columns for the object, diameter, and circumference:

A. Use a tape measure or string to measure the circumference and diameter of several different circular objects. Record your results in a table with columns for the object, diameter, and circumference.
B. Study your table. Look for patterns and relationships between the circumference and the diameter. Test your ideas on some other circular objects.
   1. Can you find the circumference of a circle if you know its diameter? If so, how?
   2. Can you find the diameter of a circle if you know its circumference? If so, how?

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Covering and Surrounding • page 75

**Problem 5.3 Exploring Area and Circumference**

A. Find as many different ways as you can to estimate the area of the pizzas. For each method, give your estimate for the area and describe how you found it.
B. Copy the table and record each pizza’s size, diameter, radius, circumference, and area in a table.

<table>
<thead>
<tr>
<th>Size</th>
<th>Diameter</th>
<th>Radius</th>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. Examine the data in the table and your strategies for finding area. Describe any shortcuts that you found for finding the area of a circle.
D. In your opinion, should the owner of the pizzeria base the cost of a pizza on area or on circumference? Explain.
In the fourth problem, students estimate the number of “radius squares” (squares with side length equal to the radius of the circle) it takes to cover a circle. This problem helps students discover the formula for the area of a circle and to understand why it makes sense. Students should find that the area of a circle is “a little bit more than 3” radius squares. With the help of the teachers, students relate “a little bit more than 3” to the number \( \pi \), and develop the area formula \( A = \pi \cdot r^2 \). Mental images such as the square embedded in a circle trigger a way for students to recall the formula for the area of a circle and to remember why the formula makes sense.

**Covering and Surrounding • page 77**

<table>
<thead>
<tr>
<th>Problem 5.4 Finding Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A portion of each circle is covered by a shaded square. The length of a side of the shaded square is the same length as the radius of the circle. We call such a square a “radius square.”</td>
</tr>
</tbody>
</table>

**A.** How many radius squares does it take to cover the circle? (You can cut out radius squares, cover the circle and see how many it takes to cover.)

**B.** Describe any patterns and relationships you see in your table that will allow you to predict the area of the circle from its radius square. Test your ideas on some other circular objects.

**C.** How can you find the area of a circle if you know the radius?

**D.** How can you find the radius of a circle if you know the area?

Homework starts on page 78.

The sequencing of this set of problems and its effectiveness is reflective of the interactions between the authors and the teachers and students at our trial sites.

**CMP: A Curriculum Co-Developed With Teachers and Students**

Developing a curriculum with a complex set of interrelated goals takes time and input from many people. As authors, our work was based on a set of deep commitments we had about what would constitute a more powerful way to engage students in making sense of mathematics. Our Advisory Board took an active role in reading and critiquing units in their various iterations. In order to enact our development principles, we found that three full years of field trials in schools was essential for each year of the materials. This feedback from teachers and students across the country is the key element in the success of the *Connected Mathematics Project* materials. The final materials comprised the ideas that stood the test of time in classrooms across the country. Nearly 200 teachers in 15 trial sites around the country (and their thousands of students) are a significant part of the team of professionals that made these materials happen. The scenarios of teacher and student interactions with the materials became the most compelling parts of the Teacher’s Guides.

Without the bravery of these teachers in using materials that were never perfect in their first versions, CMP would have been a set of ideas that lived in the brains and imaginations of the five authors. Instead, they are materials with classroom heart because our trial teachers and students made them so. We believe that such materials have the potential to dramatically change what students know and are able to do in mathematical situations. The emphasis on thinking and reasoning, on making sense of ideas, on making students responsible for both having and explaining their ideas, on developing sound mathematics habits gives students opportunities to learn in ways that can change how they think of themselves as learners of mathematics.

From the authors’ perspectives, our hope is to develop materials that play out deeply held beliefs and firmly grounded theories about what mathematics is important for students to learn and how they should learn it. We hope that we have been a part of helping to challenge and change the curriculum landscape of our country. Our students are worth the effort.