

## **Dear Family,**

The next unit in your child's mathematics class this year is ***Growing, Growing, Growing: Exponential Relationships***. This unit focuses on exponential relationships, in which a quantity grows larger or smaller at an increasing rate rather than at a constant rate.

### **UNIT GOALS**

Your child has previously studied linear growth, in which a fixed amount is repeatedly added to a beginning quantity to produce a sequence of values. Exponential growth involves patterns that are based on multiplication rather than addition. For example, in the sequence 3, 9, 27, 81, 243, ..., each term is 3 times the previous term.

The basic goal in *Growing, Growing, Growing* is for students to learn to recognize situations, data patterns, and graphs that are modeled by exponential expressions, and to use tables, graphs, and equations to answer questions about exponential patterns.

### **HELPING WITH HOMEWORK**

You can help with your child's homework and encourage sound mathematical habits as your child studies this unit by asking questions such as:

- Is the relationship between variables an example of exponential growth or decay? Why?
- How can this relationship be detected in a table, graph, or equation? What is the growth factor?
- What table, graph, or equation would model the data or the pattern in a graph relating the variables?
- How could I answer questions about an exponential situation by studying a table, a graph, or an equation of the exponential relationship?
- How does this exponential relationship compare to other relationships between variables I have studied?

In your child's notebook, you can find worked-out examples from problems done in class, notes on the mathematics of the unit, and descriptions of the vocabulary words.

### **HAVING CONVERSATIONS ABOUT THE MATHEMATICS IN *GROWING, GROWING, GROWING***

You can help your child with his or her work for this unit in several ways:

- Talk with your child about the applications that are presented in the unit and similar applications that you encounter in your daily activities.
- Have your child pick a question that was interesting to him or her and explain it to you.
- Look over your child's homework and make sure that all questions are answered and that explanations are clear.

A few important mathematical ideas that your child will learn in *Growing, Growing, Growing* are given on the back. As always, if you have any questions or concerns about this unit or your child's progress in class, please feel free to call.

Sincerely,

## Important Concepts

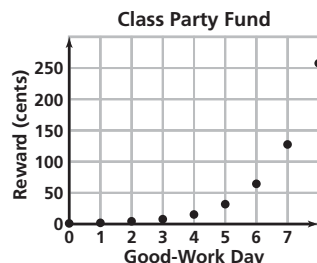
### Exponential Growth

An exponential pattern of change involves patterns that are based on multiplication and can often be recognized in a verbal description of a situation or in the pattern of change in a table of  $(x, y)$  values.

The increasing rate of growth is reflected in the upward curve of the plotted points.

## Examples

Suppose a reward is offered. At the start, 1¢ is put in a fund. On the first day, 2¢ is added; on the second day, 4¢ is added; and on each succeeding day, the reward is doubled. How much money is added on the eighth day?



Class Party Fund

Good-Work Day	Reward (cents)
0 (start)	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	?

### Growth Factor

A constant factor can be obtained by dividing each successive  $y$ -value by the previous  $y$ -value. This ratio is called the *growth factor* of the pattern.

In the example above, you multiply the previous award by 2 to get the new reward. This constant factor can also be obtained by dividing successive  $y$ -values:  $\frac{2}{1} = 2$ ,  $\frac{4}{2} = 2$ , etc.

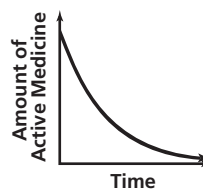
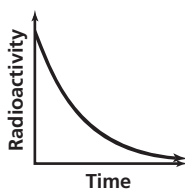
### Exponential Equations

**EXPONENTIAL GROWTH** An exponential growth pattern,  $y = a(b)^x$ , increases slowly at first but grows at an increasing rate because its growth is multiplicative. The growth factor is  $b$ .

Day	Calculation	Reward (cents)
0	1	1
1	$1 \times 2$ , or $2^1$	2
2	$1 \times 2 \times 2$ , or $2^2$	4
3	$1 \times 2 \times 2 \times 2$ , or $2^3$	8
⋮	⋮	⋮
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ , or $2^6$	256
⋮	⋮	⋮
$n$	$1 \times 2 \times 2 \times \dots \times 2$ , or $2^n$	$2^n$

On the  $n$ th day, the reward,  $R$ , will be  $R = 1 \times 2^n$ . Because the independent variable in this pattern appears as an exponent, the growth pattern is called exponential. The growth factor is the *base*, 2. The *exponent*,  $n$ , tells the number of times the 2 is a factor.

**EXPONENTIAL DECAY** Exponential models describe patterns in which the value decreases. Decay factors result in decreasing relationships because they are less than 1.



$$y = 50\left(\frac{1}{2}\right)^n$$

### Rules of Exponents

The multiplicative structure of bases leads to:

$$(b^m)^n = b^{mn}$$

$$(b^m)(b^n) = b^{m+n}$$

$$(a^m b^m) = (ab)^m$$

$$a^m / a^n = a^{m-n}$$

$$(2^3)^2 = (2 \times 2 \times 2)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$$

$$3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5 = 243$$

$$(2 \times 5)^2 = (2 \times 5) \times (2 \times 5) = (2 \times 2) \times (5 \times 5) = 2^2 \times 5^2$$

$$5^3 / 5^2 = (5 \times 5 \times 5) / (5 \times 5) = 5^{3-2} = 5^1 = 5$$

On the **CMP Parent Web Site**, you can learn more about the mathematical goals of each unit, see an illustrated vocabulary list, and examine solutions of selected ACE problems. <http://PHSchool.com/cmp2parents>